Growth and Yield Modelling for Unthinned *Acacia mangium, Acacia crassicarpa* and *Eucalyptus pellita* Plantations in Indonesia

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by

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ABSTRACT

Several species, including *Acacia crassicarpa* (Sugesty, Kardiansyah, & Pratiwi, 2015) and *Eucalyptus pellita* (Lee, 2003) have been used as alternatives for *Acacia mangium* in Indonesia due to reductions in site quality in successive rotations (Sugesty et al., 2015) or because of a root disease (Lee, 2003). Growth and yield models of these species have not been adequately researched. Some studies (Krisnawati, Wang, & Ades, 2010; Kurinobu, Arisman, Hardiyanto, & Miyaura, 2006; Lazuardi, 2009; Lumbres et al., 2015) have been carried out for *Acacia mangium* in Indonesia, but no growth and yield model has been built in the study area. This study aimed to create stand level and diameter distribution models for these species. Additional objectives were to create generalised height and diameter equations for these species and compatible taper and volume equations for *Acacia mangium.*

Data were collected from Permanent Sample Plots (PSPs) in parts of Riau Province for *Acacia crassicarpa* and *Eucalyptus pellita,* and also parts of East Kalimantan for *Acacia mangium*. Data for taper and volume equations were derived from stem analysis of 192 trees. All these species are used for pulp and paper production and no thinnings are involved in their silvicultural regimes. We used an all-possible interval approach to create mean top height, basal area, maximum diameter over bark at breast height (dbhob) and standard deviation of dbhob models. Particularly for mortality, we compared three approaches by using all-possible interval, one-year interval and longest interval approaches.

We found that the Näslund (1937) equation was the best two-parameter height–diameter model for all species. Generalised height–diameter equations were created by adding stand variables (site index, basal area/ha, stocking/ha, age and elevation) into parameters in that equation. The stand variables that affected height estimations varied among the species. A general combined variable with scaled power transformations was selected for predicting the volume of *Acacia mangium* and a four-parameter polynomial equation was chosen as the best taper equation for this species.

A von Bertalanffy–Richards polymorphic was the best equation for mean top height projections for all species and a two-parameter Schumacher polymorphic was the best equation for basal area projection for all species. A two-step regression procedure (Woollons, 1998) with a one-year interval was selected for projecting mortality, because it produced the smallest bias

compared with other approaches. Mortality equations were specific for each species. We found that a Weibull anamorphic equation was the best mortality model for *Acacia crassicarpa,* and an exponential decay anamorphic and a two-parameter Schumacher polymorphic equation were the best models for *Acacia mangium* and *Eucalyptus pellita* respectively.

The best model for estimating the standard deviation of dbhob was the von Bertalanffy–Richards polymorphic. A von Bertalanffy–Richards polymorphic was also the best model for estimating maximum dbhob for both *Acacia* species. Meanwhile, a two-parameter Schumacher polymorphic was the best model for *Eucalyptus pellita*.

Site variability and climatic factors for augmented models were elevation, mean annual temperature and mean annual rainfall. We found that elevation had an effect on mean top height for all species and on basal area for *Acacia mangium*. Meanwhile, mean annual rainfall had an effect on basal area for all species and on maximum dbhob for *Acacia mangium*. The augmented mortality models, the augmented models of maximum dbhob and standard deviation of dbhob for *Acacia crassicarpa* and *Eucalyptus pellita* were not recommended from this analysis. However, improvement for all these augmented models gave less than 5% reduction of standard error compared with their empirical models.

Diameter distributions of forest stands can be estimated using reverse three-parameter Weibull distributions by employing stand level models, maximum dbhob and standard deviation of dbhob models. Furthermore, by using volume equations and the mid-point of each diameter classes, the total volume of each dbh class can be projected. For commercial purposes, this estimation will help forest managers obtain information about commercial logs available in larger trees.

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LIST OF SYMBOLS AND ACRONYM

A	= age (years)
a, b, c, d	= parameters of an equation
Acra	= abbreviation of *Acacia crassicarpa* on graph
AD	= aggregate deviation
AIC	= Akaike information criterion
Aman	= abbreviation of Acacia mangium on graph
bh	= breast height that used in Indonesia (1.3 m)
cdf	= cumulative density function
d	= diameter at point *h* (cm)
d^2	= quadtratic diameter at point *h* (cm^2)
dbh	= diameter at breast height (cm).
dbhob	= diameter over bark at breast height (cm)
$dbhob^2$	= quadratic diameter over bark (cm^2 or m^2)
$dbhob^2 h_t$	= quadratic diameter over bark multiplied by total height (m^3)
Dmax	= maximum dbhob of a stand (cm)
do	= diameter over bark (cm)
Dstd	= standard deviation of dbhob (cm)
du	= diameter under bark (cm)
E	= Elevation *(m)*
EI	= Error Index
Epel	= abbreviation of *Eucalyptus pellita* on graph
G	= stand basal area (m^2/ha)
h	= height or length (m). In taper section, it is defined as distance up to the stem from the ground (m).
H	= dominant stand height (m). It can be MTH or PMH.
h_t	= total tree height (m)
MAE	= mean absolute error
MD	= average deviation
MEF	= model efficiency
MRES	= mean residuals

MTD = mean top diameter (cm). It is defined as the average diameter of 100 largest trees per hectare.

MTH = mean top height. It is a predicted value that derived from height and diameter equation by computing mean top diameter as an input variable.

N = stand stocking (stems/ha).

pdf = probability density function.

PMH = predominant height (m). it is defined as the actual mean height from 100 dominant tree per hectare.

R^2 = coefficient of determination

R^2a = adjusted R^2

RMS = residuals mean squares

RMSE = root mean squares error

SDI = stand-density index (stems/ha)

SEE = standard error of estimates

SI = site index at age 5 years (m)

SPT = scaled power transformation

T = mean annual temperature (°C) or age (years)

T1 or *T2* = age at certain measurement (years)

v = volume from the tip of the tree to a point h_t-*h* with *d* diameter (m^3)

V = stand volume (m^3/ha)

v_{mv} = merchantable volume of a tree (m^3)

vob = volume over bark (m^3)

vub = volume under bark (m^3)

v_t = estimated total volume through a volume function (m^3)

α = significance level

β = coefficients or parameters of an equation

λ = lambda value of scaled power transformation

CHAPTER I

Introduction

1.1. Introduction

Sustainable forest management is an essential goal that Indonesia aims to achieve because of the enormous amount of deforestation that has taken place over previous decades (FAO, 2005; Margono et al., 2012; Miettinen, Shi, & Liew, 2011). In order to minimise levels of deforestation, Indonesia has extended a moratorium regulation to postpone the issuing of new licences for harvesting natural forest and peatland areas until 2019 (Instruksi Presiden Republik Indonesia No. 6 Tahun 2017, 2017). Indonesia will therefore now need to rely on its plantation forests to fulfil the need for timber and pulp. Good management needs to be achieved to ensure a continuous wood supply from plantation forest.

Decisions made in forest management rely on information about current and future forest conditions (Avery & Burkhart, 1994). Current forest information can be obtained from an inventory whereas future forest conditions can only be predicted, due to the continuously changing state of a forest (Avery & Burkhart, 1994). Direct methods used to predict future forest conditions commonly use stand table projections that involve past growth and mortality from field observations in existing stands (Avery & Burkhart, 1994). However, stand table projections are best suited to uneven-aged, low density and immature timber stands. In addition, a stand table projection is not reliable in predicting future forest conditions over a long period (Avery & Burkhart, 1994). Another way to predict the forest stand dynamic is using an indirect method where information is inferred from the study of other stands through the use of tables, equations or computer simulation models that are referred to as growth and yield models (Avery & Burkhart, 1994).

Growth and yield modelling is essential for forest management, especially for the decision-making process to predict future forest yield and to explore silvicultural options that might influence future yield (Vanclay, 1994). It may be required for short-term or long-term periods and uneven-aged or even-aged stands as overall stand volume or volume by product and size class (Avery & Burkhart, 1994). Moreover, growth modelling is an essential tool for

calculating future harvest volumes and financial returns (Latifah, Villanueva, Carandang, Bantayan, & Florece, 2014).

The government of Indonesia obliges every forest company to create a growth and yield model. Standards and criteria for sustainable forest management in Indonesia (Perdirjen BUK P.8/2012 and the latest version P.14 /2016) require forest companies to use an increment that is derived from measurements from Permanent Sample Plots (PSPs) to be implemented in their management plans. Perdirjen BUK P.7/2014 also mentions that every plantation forest must have a yield table if the plantation is in its second rotation.

Current long-term management plans in some plantation forests in Indonesia use assumptions of mean annual increment (MAI) to calculate future yield. Typically, these are derived from research by just a few companies and are then widely applied to other companies. For example, some plantation forests of *Acacia sp*. commonly use an assumption of MAI at first rotation of 20 m^3/ha/year due to non-optimal land clearing and 30 m^3/ha/year for second rotation. Others use an assumption that MAI for *Acacia sp*. is 25 m^3/ha/year. This assumption is too unreliable to generalise across some forests in Indonesia, which might vary in ecosystem type, stand density, site quality, silvicultural treatments, soil type and other factors that might influence the growth of a species.

Indonesia ranked ninth in the world pulp industry and sixth in the world paper industry in 2016 (Himawan, 2016) with *Acacia mangium, Acacia crassicarpa*, and *Eucalyptus pellita* as the main species used for production. In some plantation forests, *Acacia crassicarpa* (Sugesty, Kardiansyah, & Pratiwi, 2015) and *Eucalyptus pellita* (Lee, 2003) have been used recently as an alternative species for *Acacia mangium* due to reductions in site quality in the following rotations (Sugesty et al., 2015) or because of root disease (Lee, 2003). Therefore, growth and yield of these species has not been adequately researched. Unlike the two previous species, growth and yield models of *Acacia mangium* have been built for some regions in Indonesia. However, according to Indonesian regulations (Perdirjen BUK P.14 /2016 and Perdirjen BUK P.7/2014) every forest company is required to have information about the growth and yield of its forest.

1.2. Objective

This study aimed to create growth and yield models for *Acacia mangium* in East Kalimantan, and *Acacia crassicarpa* and *Eucalyptus pellita* in Riau Province. The specific objectives were:

1. to develop volume equations for *Acacia mangium* and height–diameter equations for all species as a part of a preliminary analysis.
2. to develop growth (mean top height and basal area) and mortality models for *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita* at a stand-level.
3. to generate yield equations based on growth models for those species.
4. to create diameter distribution models for those species.

1.3. Study Area

1.3.1. Location

The study was carried out for two plantation forest companies: PT. Arara Abadi (AA) and PT. Fajar Surya Swadaya (FSS) in two provinces in Indonesia: Riau and East Kalimantan province. Riau is a province of Indonesia that has the largest area of production forest (4.6 million hectares) in Sumatra Island and which also contributes about 7% of the total volume of the production of forests in Indonesia (Kementrian Lingkungan Hidup dan Kehutanan, 2014). Of the 5.5 million hectares of forested area in this province (Kementrian Lingkungan Hidup dan Kehutanan, 2014), 3.9 million hectares are located in wetland areas (Balai Besar Penelitian dan Pengembangan Sumberdaya Lahan Pertanian, 2011). Approximately 19 % of the log products in this province are used for fabricating pulp either for export or domestic use (Dinas Kehutanan Provinsi Riau, 2014). Meanwhile, East Kalimantan contains about 9% of the total production forest area in Indonesia (Kementrian Lingkungan Hidup dan Kehutanan, 2014), with pulp production only 0.23% of the total log products in this province (Dinas Kehutanan Provinsi Kalimantan Timur, 2015) .

AA is located in Riau Province, between 100° 42′ E and 102° 54′ E, and 00° 00′ N and 01° 39′ N (PT Arara Abadi, 2011), particularly in Bengkalis Regency, Kampar Regency, Pekanbaru Regency, Pelalawan Regency and Siak Regency (Wibisono et al., 2015). It received a permit for plantation forest from the Ministry of Forestry in 1996, by the Minister of Forestry Decree No. 743/Kpts-II/1996 (Wibisono et al., 2015) for an area of 296,262 ha. The species predominantly planted by this company are *Acacia crassicarpa* and *Eucalyptus pellita* and they are grown on 5-year rotations. The size of all PSP plots established by this company is 0.02 ha.

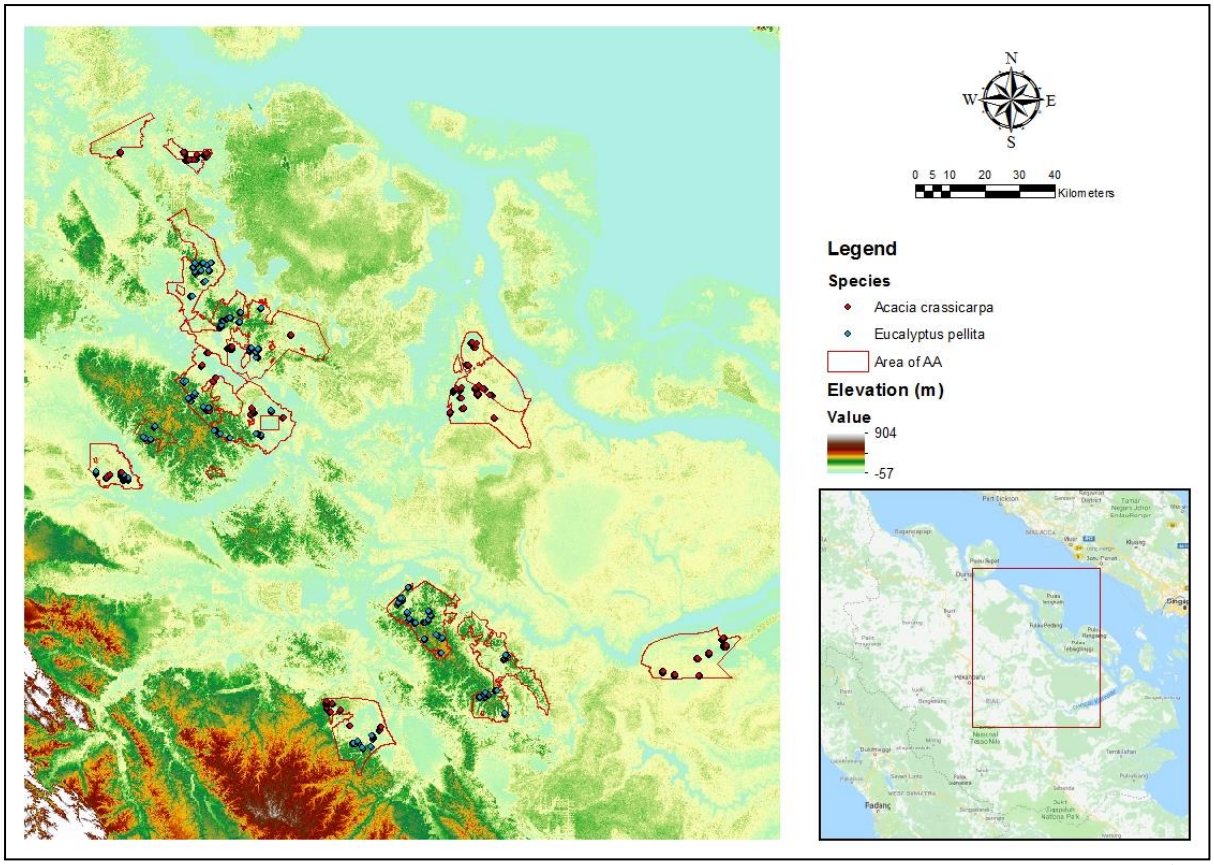

Figure 1 PSP distribution at AA area in Riau Province (*Acacia crassicarpa* and *Eucalyptus pellita*)[1]

The climate in AA can be categorised as type A according to the Schmidt and Fergusson classification, with Q value (the percentage of dry months relative to wet months) = 14%. Based on Bandar Udara Simpang Tiga Meteorological Station, the average rainfall per month in AA area was 216.56 mm with average humidity of 76% and average temperature 27.22°C (PT Arara Abadi, 2011). Instead of thinning and pruning, this company does singling to cut the competitor stem in multileader trees at age 4–6 months. However, the PSP plots did not receive this treatment.

FSS is located in East Kalimantan, specifically in Penajam Paser Utara Regency and Paser Regency. In 1997, FSS received a permit to manage plantation forest by the Ministry of Forestry Decree No. 383/Kpts-II/97 for an area of 66,659 ha (Dinas Kehutanan Provinsi Kalimantan Timur, 2016). FSS plants *Acacia mangium* for pulp production with rotation age of 6 years. The size of all PSP plots established by this company is 0.05 ha. This company

[1] Sources : USGS, Google, AA

does not implement thinning and pruning, but they do singling (cutting competitor stems at early growth) in their area, including in PSP plots.

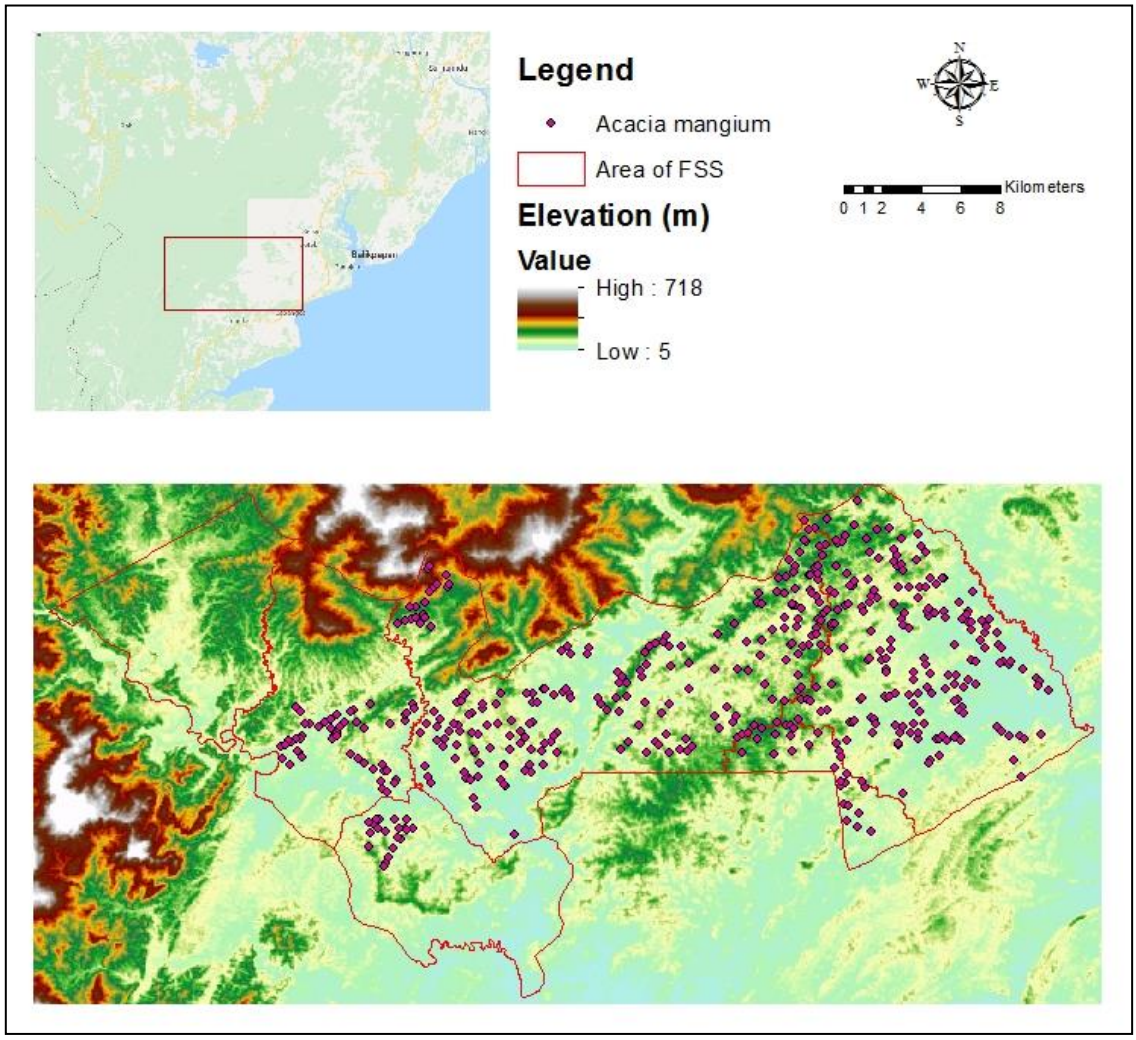

Figure 2 PSP distribution at FSS area in East Kalimantan Province (*Acacia mangium*)[2]

1.3.2. Species

Acacia mangium Wild originates from Queensland, Australia, the southwestern part of New Guinea and the Molucca Island in the eastern part of Indonesia (National Research Council, 1983). The new leaves of the juvenile plants of this species look similar to those of species from subfamily Mimosoideae. Nonetheless, the leaf stalk and main axis of each component leaf becomes flattened and transforms into a phyllode after a few weeks (Krisnawati, Kallio, & Kanninen, 2011). This species is easily adapted to a varied range of environments and soil conditions, even in low-nutrient soil, acidic soil and degraded sites or in areas that are

[2] Sources : USGS, Google, FSS

dominated with *Imperia cylindrica* (National Research Council, 1983). However, this species is intolerant of shade and saline conditions (National Research Council, 1983).

Acacia crassicarpa A. Cunn ex. Benth is a native tree from Australia, Indonesia and Papua New Guinea that is commonly found in warm to hot, humid and subhumid zones in lowland tropics, especially behind beaches, coastal plains and foothills (World Agroforestry Centre, 2009a). This species visually has a similar appearance to *Acacia mangium*. The name "crassicarpa" is derived from its thick pod (World Agroforestry Centre, 2009a). In contrast with *Acacia mangium*, this species is tolerant of salt spray and soil salinity (World Agroforestry Centre, 2009a).

Eucalyptus pellita F. Muell is a native tree from Australia (World Agroforestry Centre, 2009b) and is commonly planted in some regions in Indonesia as an alternative species to replace *Acacia mangium* plantations which have been severely affected by root disease (Lee, 2003). This species is frost-resistant, it grows fast in humid, subhumid and tropical lowland zones and requires uniform summer rainfall (World Agroforestry Centre, 2009b). This species produces oil from its leaves and can be used as an ant- and pest-repellent (World Agroforestry Centre, 2009b).

Figure 3 True leaves of *Acacia crassicarpa* (left); phyllode of *Acacia mangium* (middle); *Eucalyptus pellita* (right)

CHAPTER II

Literature Review

2.1. Forest Growth and Yield Modelling

The term "growth" is defined as the increment over a given period of time, while "yield" is defined as the sum of annual increment or total amount available for harvest at a given time (Avery & Burkhart, 1994). Burkhart and Tomé (2012) categorised forest modelling approaches into whole-stand models, diameter distribution models, size class models and individual trees. The whole-stand approach commonly estimates volume, basal area, and/or the number of trees per unit area by using age, site index, site quality, and stand density as predictor variables (Burkhart & Tomé, 2012). The diameter distribution approach is a variation of whole-stand models, employing a probability-density function to represent the number of trees of different diameters at breast height (dbh) classes (Burkhart & Tomé, 2012). Size class models usually involve projecting stand tables (numbers of trees by diameter class) via an equation for upgrowth and ingrowth or employing a matrix model (Burkhart & Tomé, 2012). Unlike whole-stand models, individual-tree models use individual trees as the modelling entity (Burkhart & Tomé, 2012).

Growth and yield models are commonly classified into stand-level models, distance-dependent individual-tree models, and distance-independent individual-tree models (Munro, 1974). Whole-stand models are simple, fast in computation and also have the capability to utilise conventional inventory information (Munro, 1974). The disadvantage of this model is a lack of information about individual trees (Munro, 1974).

Desirable logical properties of functions used for growth and yield models are compatibility of growth and yield, logical consistency of the functions, path invariance and asymptotes (Clutter, 1983). Compatibility means that there is an integral (yield) and derivative (growth) relationship, or the yield model can be attained by summarising the estimated growth through an appropriate growth period (Clutter, 1963). Given an example equation:

$$Y_2 = \left(\frac{T_1}{T_2}\right) Y_1 + \alpha_0 \left(1 - \frac{T_1}{T_2}\right), \qquad \text{(Equation 2. 1)}$$

logical consistency means that if $T_1 = T_2$ then $Y_1 = Y_2$. Path invariance implies that when we predict Y_3 from Y_1 in a single step, this should give the same result as when we predict Y_2 from Y_1, followed by Y_3 from Y_2. Asymptote means that as T_2 approaches infinity, Y_2 approaches α_0 and the model provides an upper asymptote for future yield (Clutter, 1983).

Table 1 *Some sigmoid and nonsigmoid functions commonly used in forest modelling (Burkhart & Tomé, 2012)*

No.	Author or designation	Yield equation (integral form)	Growth equation (differential form)	Note
1	Hyperbola	$Y = a_0 + a_1^{1/t}$	$\frac{dY}{dt} = -a_1^{1/t^2}$	Nonsigmoid
		$Y = a_0 + a_1^{1/t} + a_2^t$	$\frac{dY}{dt} = a_2 - a_1^{1/t^2}$	Nonsigmoid
		$Y = a_0 + a_1 \log t$	$\frac{dY}{dt} = a_1 - \frac{1}{t}$	Nonsigmoid
2	Linear-logarithmic	$Y = a_0 + a_1 t^{a_2}$	$\frac{dY}{dt} = \frac{a_2}{t}(Y - a_0)$	Nonsigmoid
		$Y = (a_0 + a_1 t)^{a_2}$	$\frac{dY}{dt} = \frac{a_0 a_2 Y}{a_0 + a_1 t}$	Nonsigmoid
3	Exponential	$Y = a_0 + a_1 e^{a_2 t}$	$\frac{dY}{dt} = a_2(a_0 - Y)$	Nonsigmoid
4	Freese	$Y = a_0 t^{a_1} + a_2^t$	$\frac{dY}{dt} = Y\left(\log a_2 + \frac{a_1}{t}\right)$	Nonsigmoid
5	Hossfeld I	$Y = \frac{t^2}{a_0 + a_1 + a_2 t^2}$	$\frac{dY}{dt} = Y^2\left(\frac{2a_0 + a_1 t}{t^3}\right)$	Nonsigmoid
6	Korsun (logarithmic parabola)	$Y = a_0 t^{a_1 + a_2 \log t}$	$\frac{dY}{dt} = \frac{Y}{t}(a_1 + 2a_2 \log t)$	Nonsigmoid
7	Schumacher	$Y = Ae^{-\frac{k}{t}}$	$\frac{dY}{dt} = Y\frac{k}{t^2}$	Sigmoid
8	Johnson–Schumacher	$Y = Ae^{-\frac{k}{t+a}}$	$\frac{dY}{dt} = Y\frac{k}{(t+a)^2}$	Sigmoid
9	Lundqvist–Korf	$Y = Ae^{-\frac{k}{t^m}}$	$\frac{dY}{dt} = mY\frac{k}{t^{m+1}}$	Sigmoid
10	Monomolecular	$Y = A(1 + ce^{-kt})$	$\frac{dY}{dt} = k(A - Y)$	Sigmoid
11	Logistic	$Y = \frac{A}{(1 + ce^{-kt})}$	$\frac{dY}{dt} = \frac{k}{A}(AY - Y^2)$	Sigmoid
12	Gompertz	$Y = Ae^{-ce^{-kt}}$	$\frac{dY}{dt} = kY\,ln\frac{A}{Y}$	Sigmoid
13	von Bertalanffy-Richards	$Y = A(1 + ce^{-kt})^{-\frac{1}{1-m}}$	$\frac{dY}{dt} = \frac{kY}{1-m}\left[\left(\frac{A}{Y}\right)^{1-m} - 1\right]$	Sigmoid
14	Hossfeld/Log logistic	$Y = A\frac{t^k}{c + t^k}$	$\frac{dY}{dt} = k\frac{Y}{t}\left(1 - \frac{Y}{A}\right)$	Sigmoid

2.1.1. Growth Interval

Modelling forest growth usually involves yield form and difference form data. When the yield form equation is $Y = f\ (t)$, the generic equation of difference form is $Y_2 = f\ (Y_1, t_1, t_2)$. Yield form data are commonly used for modelling initial growth during the plantation establishment stage and difference type data are widely used for modelling older forests (Methol, 2001). The latter form can be divided into overlapping and nonoverlapping intervals. From n measurements, the nonoverlapping type only has $n - 1$ data. Conversely, the overlapping type can have all-possible intervals that are derived from PSP measurements (Methol, 2001). For instance, a datum from three measurements could have not only $n - 1$ interval data ($T_1 - T_2$ and $T_2 - T_3$) but also one additional datum ($T_1 - T_3$) . Moreover, these latter datasets include the longest interval (Borders, Bailey, & Clutter, 1988).

The advantage of using all-possible intervals to fit a difference model is that the developed model is likely to be less biased when projecting long growth intervals than models fitted with only short intervals (Lee, 1998). However, the amount of autocorrelation may increase for these intervals (Borders et al., 1988). Woollons (1998) used a constant one-year interval to avoid the bias that may occur in the two-step regression of mortality that he developed.

2.2. Growth and Yield of *Acacia crassicarpa*, *Acacia mangium* and *Eucalyptus pellita*

2.2.1. Growth and Yield Models of *Acacia sp.*

Krisnawati et al. (2011) collected information from studies of the growth rates of *Acacia mangium* in different regions in Indonesia. They found that the MAI of diameter at breast height (dbh) ranged from 1.4 to 7.3 cm/year and the MAI of height ranged from 1.8 to 5.8 m/year. Mean diameter increased rapidly up to 15 cm for stands less than three years old, and after five years, the trees were growing more slowly and diameter became steady at approximately 25 cm by the age of eight years (Krisnawati et al., 2011). The height of this species was found to reach 10 to 15 m in the first two to three years, and grow faster up to 25 m at five years, after which they levelled off (Krisnawati et al., 2011).

A growth model of *Acacia mangium* under different stand densities has been created by Kurinobu, Arisman, Hardiyanto, and Miyaura (2006) in South Sumatra, Indonesia. The data were derived from 51 permanent plots each of 0.1 ha. The initial spacing in these plots was 4 m by 2 m and they created a simulation of the spacing that is commonly used in Indonesia: 2 m

by 3 m and 3m by 3m, with thinning assumed at age five and seven years. They found that stand development of *Acacia mangium* could be predicted by using a growth model that consisted of a polymorphic site index equation, a reciprocal equation of the diameter–density effect and a self-thinning equation. Moreover, Lazuardi (2009) created growth and yield models for *Acacia mangium* in South Borneo, Indonesia. He found that the Chapman–Richards model was the best model for an unthinned stand with spacing 3 m by 3 m and the Lundqvist–Korf model was the best model for an unthinned stand with spacing 4 m by 2 m.

Lumbres et al. (2015) conducted a study to create a height–dbh model for *Acacia mangium* in Central Borneo, Indonesia, and found that the Weibull model was the best fit to explain the relation between dbh and height of this species. They used 2,394 trees of *Acacia mangium* to create a model and found that the mean dbh of this species was 17.79 cm and the mean height was 17.02 m. Unfortunately, information about average ages was not reported in that study.

Siregar (2008) created a volume table for *Acacia crassicarpa* in wetland area plantation forest in Jambi, Indonesia. Based on the assumption that mean top height is not relevant to determine site quality in wetland areas (due to unstable ground surfaces that lead to trees falling in strong winds), he found that stand density could be used to determine the site quality of wetland areas. However, his study can be applied only to a stand with 1,666 trees/ha. Patricia (2006) found that the Chapman–Richards model was the best model for predicting the height of *Acacia crassicarpa* in wetland areas of plantation forest in Jambi, Indonesia.

2.2.2. Growth and Yield Model of *Eucalyptus sp.*

Lumbres et al. (2015) conducted a study to create a height–dbh model for *Eucalyptus pellita* in Central Borneo and found that the Lundqvist–Korf model was best fitted to explain the relationship between dbh and height of this species. In addition, they found that *Eucalyptus pellita* tended to have a higher total height compared with *Acacia mangium* for trees that have a dbh of more than 28 cm.

Latifah et al. (2014) conducted a study to predict growth for five species groups of *Eucalyptus* in North Sumatra, Indonesia: *Eucalyptus* hybrid, *Eucalyptus grandis*, *Eucalyptus pellita*, *Eucalyptus urophylla*, and mixed *Eucalyptus*. They used 650 plots that consisted of 106 PSP plots and 544 inventory plots, divided into 65 plots for validation (10%) and the rest for creating the model. They found that each *Eucalyptus* species required a specific model to predict growth and yield, with the *Eucalyptus pellita* model having slower growth after the age of five years

compared with the others. Basal area, height, age, site index, spacing, depth of soil, rainfall and soil group were correlated with growth of these species.

2.3. Mortality Approaches

Tree mortality is the most difficult process to model, yet it is important in forest growth and yield projections (Weiskittel, Hann, Kershaw, & Vanclay, 2011). It can be categorised into regular or irregular, but sometimes the distinction between these types may be blurred (Weiskittel et al., 2011). Regular mortality is defined as being caused by competition-induced suppression and it is also known as density-dependent mortality (Weiskittel et al., 2011). Irregular mortality, however is caused by external or catastrophic factors such as wind, fire, insects, disease and many more (Weiskittel et al., 2011).

A mortality projection of forest trees is usually the inverse of sigmoid shape, describing the decreasing of stocking over time (Istvan, 1931). Various approaches have been used to predict mortality, one of them being a two-step regression with a constant period of an interval, proposed by Woollons (1998), and another which advocated the use of the longest interval (Lee, 1998; Methol, 2001; Zhao, 1999). The notion of these two methods was derived from the stagnancy of stocking that may occur over several years. If we exclude data that indicates that no mortality occurred in a period, the model tends to overestimate (Woollons, 1998). In order to avoid that, Woollons (1998) suggested using probability of mortality to adjust the equations. On the other hand, Methol (2001) advised using the longest interval for mortality projection, since the chance of stocking reduction increases with increasing period length. Probability equations (Equation 2.2) and adjusted mortality equations (Equation 2.3) that were proposed by Woollons (1998) are shown below.

$$p = \frac{1}{(1+exp(-(\beta_0 + \beta_1 X_1 + \ldots + \beta_n X_n)))} \qquad \text{(Equation 2. 2)}$$

$$N_{adj2} = N_1 - p\left(N_1 - \widehat{N}_2\right) \qquad \text{(Equation 2. 3)}$$

where

p = probability of stand death

β_n = parameter

X_n = set of explanatory variables (*N, T,* or perhaps *SI*)

N_{adj2} = adjusted live stems/ha at time T_2

N_1 = live stems/ha at the commenced of period

$\widehat{N}_2$ = live stems/ha at time T_2, estimated by mortality projection equation

2.4. Diameter Distribution Models

Diameter distribution models are a bridge between stand-level models and individual-tree models (Husch, Beers, & Kershaw, 2003; Nord-Larsen, 2006). Two approaches to generate this kind of model are known as the parameter prediction approach and parameter recovery approach. The first method employs the current parameters about the stand (basal area, density, volume, etc.) to directly predict the future parameters of the diameter distribution model (Husch et al., 2003). The second approach uses estimates of the moments of distribution derived from stand parameters (Husch et al., 2003).The main advantage of the second approach is that the distribution is mathematically compatible with the stand average model (Frazier, 1981). Reynolds, Burk, and Huang (1988) found that the parameter recovery approach gave a better result than parameter prediction method. They also found that their goodness-of-fit test was not designed to show the parameter error of their model and suggested using an error index instead (Reynolds et al., 1988).

The most commonly applied of the statistical distributions to generate diameter distribution models is a three-parameter distribution by Weibull, which is defined by a probability density function (Clutter, 1983). The Weibull distribution has some desirable properties that are useful for biological models, such as simplicity of algebraic manipulation and an ability to assume a diversity of curve shapes (Bailey & Dell, 1973). Moreover, the Weibull distribution is much easier to apply than a normal distribution due to its closed form and its analytic expression of the cumulative distribution function (Clutter, 1983). The probability distribution function (*pdf*) for the three-parameter Weibull distribution (Equation 2.4) and cumulative density function (*cdf*) for the three-parameter Weibull distribution (Equation 2.5) are:

$$f(X) = \frac{c}{b}\left(\frac{X-a}{b}\right)^{c-1} \exp\left[-\left(\frac{X-a}{b}\right)^{c}\right], \qquad (a \le X < \infty)$$

$$= 0, \text{otherwise} \qquad \text{(Equation 2. 4)}$$

$$F(X) = 1 - exp\left[-\left(\frac{X-a}{b}\right)^{c}\right] \qquad \text{(Equation 2. 5)}$$

where

X = tree dbh,

a = location parameter,

b = scale parameter,

c= shape parameter.

A three-parameter Weibull distribution usually employs minimum diameter as a location parameter. Naturally, small trees in a stand are significantly affected by genetics, the microsite and silvicultural treatments, and it can be hard to assess whether the suppress stems are alive or dead (Kuru, Whyte, & Woollons, 1992). On the other hand, maximum diameter usually shows a strong growth pattern and is much more strongly responsive to changes in stocking than minimum diameter is (Kuru et al., 1992). The approach of using maximum diameter is known as the three parameters of reverse Weibull distribution. The probability density function of the three parameters of reverse Weibull distribution is

$$f(X) = \frac{c}{b}\left(\frac{a-X}{b}\right)^{c-1} exp\left[-\left(\frac{a-X}{b}\right)^{c}\right], \qquad \text{(Equation 2. 6)}$$

with the cumulative density function:

$$F(X) = exp\left[-\left(\frac{a-X}{b}\right)^{c}\right] \qquad \text{if } -\infty \le X \le a$$

$$F(X) = 1, \qquad \text{if } X \ge a \qquad \text{(Equation 2. 7)}$$

where

X = tree dbh,

a = location parameter,

b = scale parameter,

c= shape parameter.

2.5. Stand Volume

Forest managers need to estimate how much volume they will get at harvesting time. From an economic point of view, total volume is an essential part of a stand model that will help in the decision-making process (Gadow & Hui, 1999). While the volume of individual trees is calculated from dbh, height and sometimes form factor, stand volume is derived from basal area, dominant height and sometimes stand age (Gadow & Hui, 1999). Some equations that are commonly used to generate stand volume are given in Equations 2.8 to 2.10:

$$V = G(\alpha_0 + \alpha_1 H) \qquad \text{(Equation 2. 8)}^3$$

$$V = \alpha_0 + \alpha_1 H + \alpha_2 G + \alpha_3 GH \qquad \text{(Equation 2. 9)}^3$$

$$V = \alpha_0 + \alpha_1 GH \qquad \text{(Equation 2. 10)}$$

where

V = stand volume (m^3/ha)

G = stand basal area (m^2/ha)

H =dominant height (m)

α_n = model parameters

2.6. Model Evaluation

Model evaluation is an important step in forest modelling to avoid unreliable predictions. An evaluation is also designed to give information about model behaviour and its predictive ability to help the forest managers to decide whether or not to use it (Soares, Tomé, Skovsgaard, & Vanclay, 1995). When evaluating a model, rejection or approval of a null hypothesis does not strictly mean that the model is wrong or right (Soares et al., 1995). When the alternative hypothesis is accepted, it means that this model is the best when compared with others models (Soares et al., 1995). Conversely, when the alternative hypothesis is rejected, it does not mean that a model is wrong and cannot be applied (Soares et al., 1995). It means that further steps need to be taken to inspect where and how the model failed and carry out more analysis to improve the model (Soares et al., 1995).

There are many ways to evaluate a model; however, there is no strict guideline about how to do it. Overall, evaluation models can be divided into qualitative and quantitative tests of a

[3] (Vanclay, 1980)

model (Gadow & Hui, 1999; Soares et al., 1995), or model criticism and model benchmarking (Vanclay, 1994; Weiskittel et al., 2011). Vanclay (1994) suggested using the term model criticism and benchmarking instead of quantitative and qualitative evaluation. Model criticism (also known as the qualitative test) is carried out to criticise the logical and biological realism of a model with or without separating data from model construction. Model benchmarking comprises statistical tests and using data independent from that used for model construction (Soares et al., 1995).

Evaluation procedures for forest growth models (Soares et al., 1995) are listed below.

1. Inspect the models and their components for logical structure and biological realism (e.g., (Bossel, 1991; Hamilton Jr, 1990; Oderwald & Hans, 1993) to guarantee that they:
   a. are parsimonious and biologically realistic,
   b. agree with existing theories of forest growth, and
   c. predict sensible responses to management actions.
2. Ascertain the statistical properties of the models in relation to data (e.g. (Bates & Watts, 1988; Ratkowsky, 1983) including:
   a. error (i.e., additive, multiplicative, independence, etc.)
   b. estimation properties of model parameters in equation functions.
3. Characterise error (e.g., Gertner, 1987; Mowrer, 1991; Power, 1993; Reynolds Jr, 1984) in terms of:
   a. magnitude,
   b. residuals (distribution, dependencies on initial stand condition and projection length),
   c. confidence intervals and critical errors,
   d. contributions by each model component to total error,
   e. propagation of error through the model.
4. Test, using statistical approaches (D'Agostino & Stephens, 1986; Gregoire & R. Reynolds, 1988; Mayer & Butler, 1993; Power, 1993; Reynolds et al., 1988) for:
   a. bias and precision of models and its components,
   b. goodness-of-fit of estimated size distributions,
   c. patterns in, and distribution of residuals,
   d. correlation between components over time.
5. Determine the component of a model that has the greatest impact on predictions by conducting a sensitivity analysis.

The essential notions of a model evaluation are bias and precision, which determine the accuracy of predictions (Gadow & Hui, 1999). Four criteria in model evaluation are mean residual (also known as mean bias or bias), which describes the directional magnitude of average model bias (underestimate or overestimate), absolute mean residual (also known as mean absolute deviation or mean absolute error), the root mean square error, and the model efficiency, which are the indices of model precision (Gadow & Hui, 1999). Methol (2001) used standard error of estimates (SEE) to measure precision.

Mean residual (MRES) $= \frac{\Sigma(Y_i - \hat{Y}_i)}{n}$ (Equation 2. 11)

Mean absolute error (MAE) = $\frac{\Sigma|Y_i - \hat{Y}_i|}{n}$ (Equation 2. 12)

Root mean square error (RMSE) = $\sqrt{\frac{\Sigma\,(Y_i - \hat{Y}_i)^2}{n}}$ (Equation 2. 13)

Model efficiency (MEF) = $\frac{\Sigma\,(Y_i - \hat{Y}_i)^2}{\Sigma\,(Y_i - \bar{Y})^2}$ (Equation 2. 14)

Standard error of estimates (SEE) = $\sqrt{\frac{\Sigma\,(Y_i - \hat{Y}_i)^2}{n-p}}$ (Equation 2. 15)

Clutter (1983) mentioned the indicator of "goodness" in candidate regressions. This indicator includes a model that has large values of coefficient of determination (R^2), adjusted coefficient of determination (R_a^2), and a small value of residual mean square. However, there are some limitations in R^2 and R_a^2 such as that models with the same R^2 and R_a^2 can have enormously different fits; they also tend to increase when the parameters are added to the model, and they may have an unexplained random error in some of the data (Weiskittel et al., 2011). Model efficiency is analogous with R^2 (Gadow & Hui, 1999).

Residuals mean square (RMS) = $\frac{\Sigma(Y_i - \hat{Y}_i)^2}{n-p}$ (Equation 2. 16)

Coefficient determination (R^2) = *1*- $\frac{\Sigma(Y_i - \hat{Y}_i)^2}{\Sigma\,(Y_i - \bar{Y})^2}$ (Equation 2. 17)

Adjusted coefficient determination (R_a^2) = $1 - (1 - R^2)\frac{(n-1)}{(n-p)}$ (Equation 2. 18)

Spurr (1952) suggested a good model should have aggregative deviation (AD) at range ±1 % and average deviation (MD) ±10%. AD and MD are commonly used in Indonesia (Isnaini,

2011; Patricia, 2006; Sufiana, 2017; Utomo, 1995) when comparing models using a validation dataset. In addition, researchers are commonly looking for a model that has a small root mean square error (RMSE) and has no significant differences between predicted and actual values (Methol, 2001; Spurr, 1952). Flather (1996) suggested using AIC (Akaike's Information Criterion) to select the best model from some best candidate models. The preferable model is the model with the lowest AIC.

$$\text{Aggregative deviation (AD)} = \left(\frac{\Sigma \hat{Y}_i - \Sigma Y_i}{\Sigma Y_i}\right) \quad x\ 100\ \% \qquad \text{(Equation 2. 19)}$$

$$\text{Average deviation (MD)} = \left(\frac{\Sigma \left(\frac{\hat{Y}_i - Y_i}{Y_i}\right)}{n}\right) \quad x\ 100\ \% \qquad \text{(Equation 2. 20)}$$

$$\text{AIC} = n\ log\left(\frac{RSS}{n}\right)^2 + 2K \qquad \text{(Equation 2. 21)}$$

where:

n = *number of* observations,

p = numbers of parameters,

Y_i = actual value of Y for observation i,

$\hat{Y}_i$ = predicted value of Y for observation i,

$\bar{Y}$ = mean of actual value

RSS = residual sum of squares

K = numbers of parameters +1

The assumptions of errors in regression analysis are that they are independent, have zero mean, a constant variance and follow a normal distribution. The residuals should display the tendencies that tend to confirm those assumptions (Draper & Smith, 1966). Residual plots allow assessment of bias. A desirable residual plot is shown in Figure 4 (a), and other plots would indicate an abnormality (Draper & Smith, 1966).

In Figure 4, plot (b) shows that the variance is not constant but increases with the predicted value (heteroscedasticity) and is unbiased. It needs a weighted least squares method or transformation before conducting regression analysis (Draper & Smith, 1966). Plot (c) explains the error in the analysis: the departure from the fitted equation is systematic negative residuals correspond to low fitted values (positive residuals to high fitted values). It can be caused by wrongly omitting a β_0 in the model (Draper & Smith, 1966). Plot (d) exhibits an inadequate

model. It may need a transformation of actual value before analysis or it may need an extra term in the model (e.g., square or cross product term) (Draper & Smith, 1966).

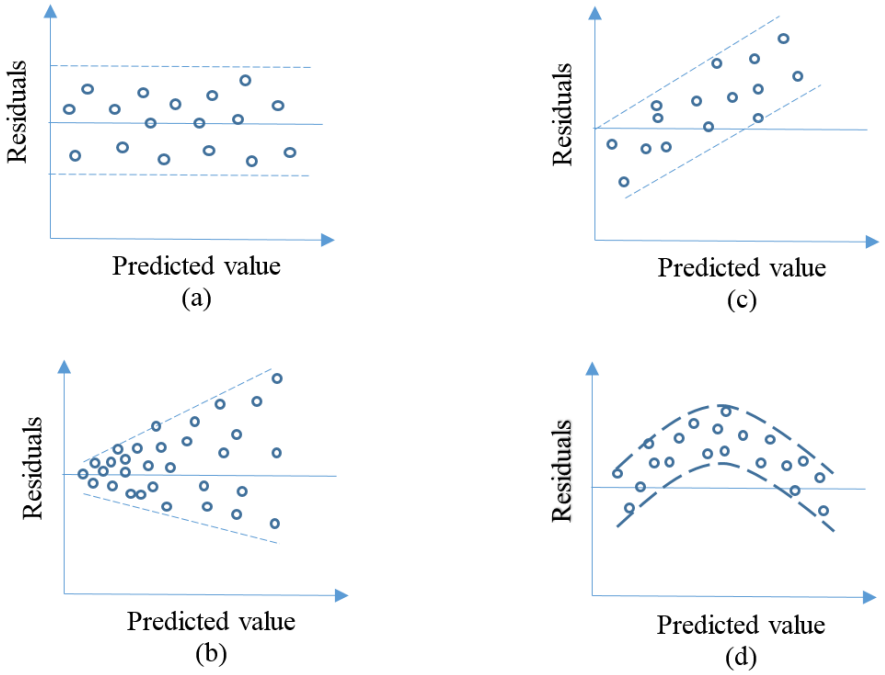

Figure 4 Residual plots of a model (Draper & Smith, 1966)

CHAPTER III

Research Methods

3.1. Data Collection

Data for this study were divided into three categories: data for developing the models, data for validating the models, and supporting data. PSP datasets for each species were used to create models and for validation of the models. Other data that were required to support this study included the history of each compartment/plot (e.g., silviculture treatments, weed control, fertilisation, thinning/unthinned), maps of soil type in plot location, maps of PSP locations and site characteristic of each plot (e.g., soil type, altitude, wet/dry land). Climatic data such as temperature and rainfall were extracted from the Bioclim database (Hijmans, Phillips, Leathwick, & Elith, 2017) using the coordinates of the PSPs. Altitude, annual precipitation and annual mean temperature are shown in Figures 5 to 9. The growth trajectories of mean top height (MTH) and basal area are shown in Figures 10 to 12.

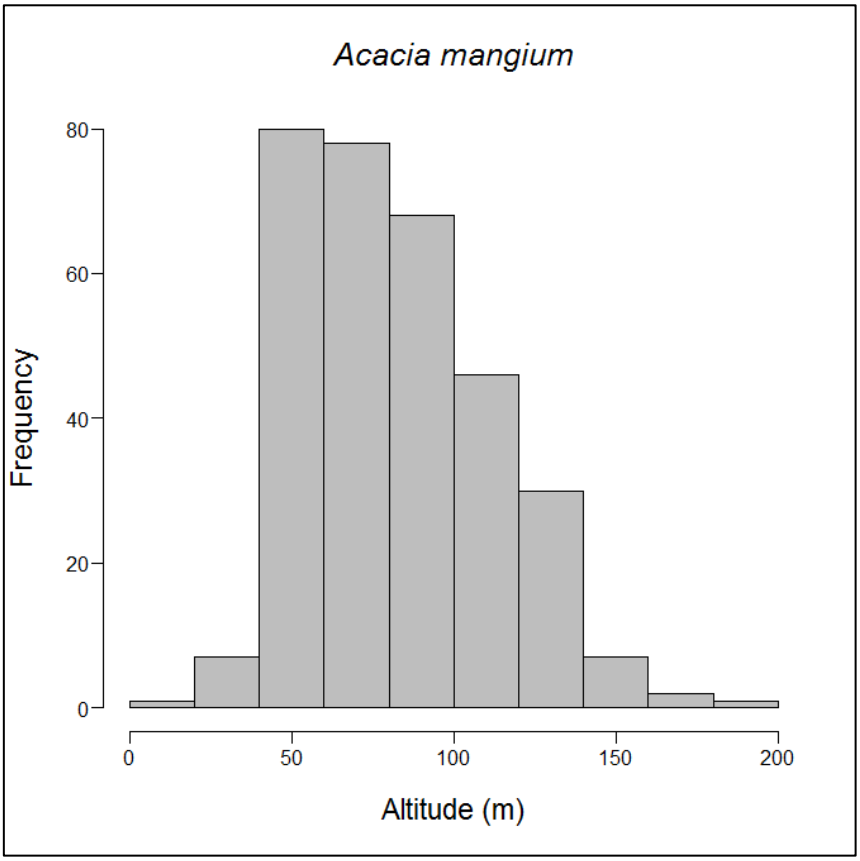

Figure 5 Elevation data of *Acacia mangium*

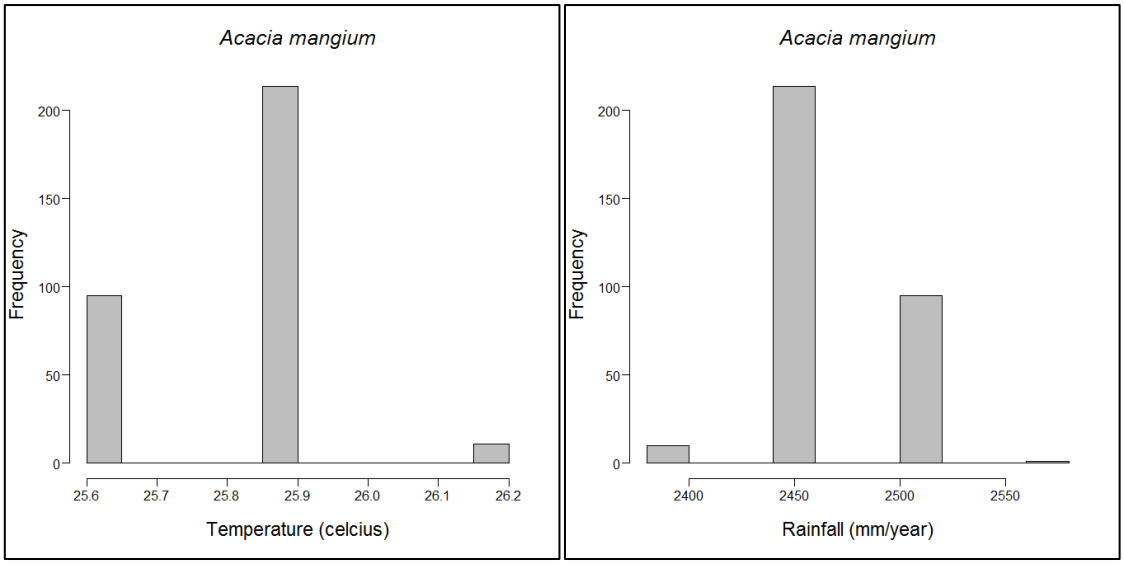

Figure 6 Annual mean temperature (left) and annual precipitation (right) of *Acacia mangium*

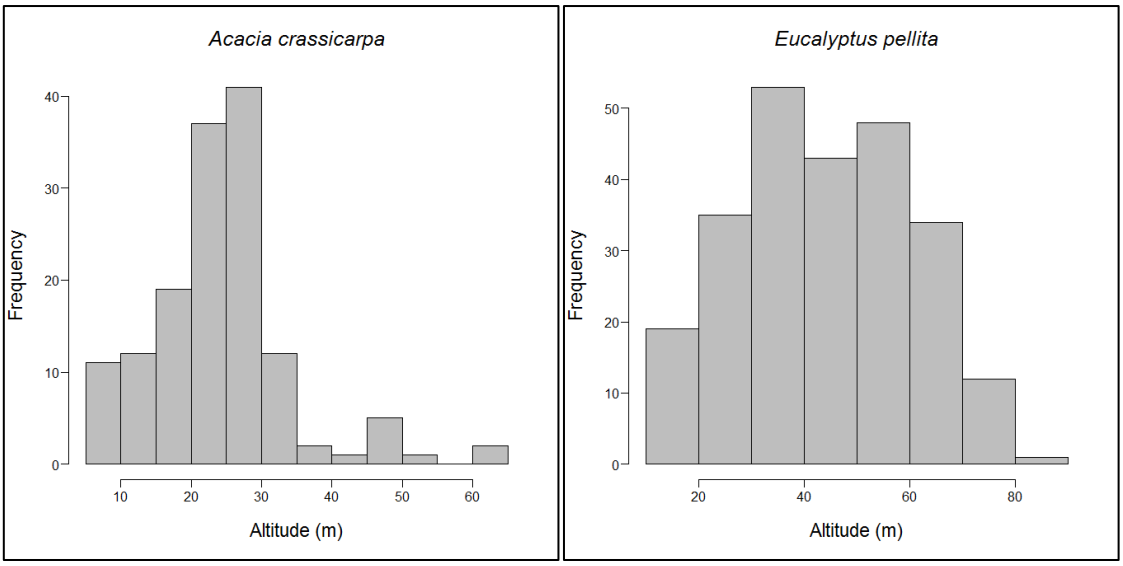

Figure 7 Elevation data: *Acacia crassicarpa* (left) and *Eucalyptus pellita* (right)

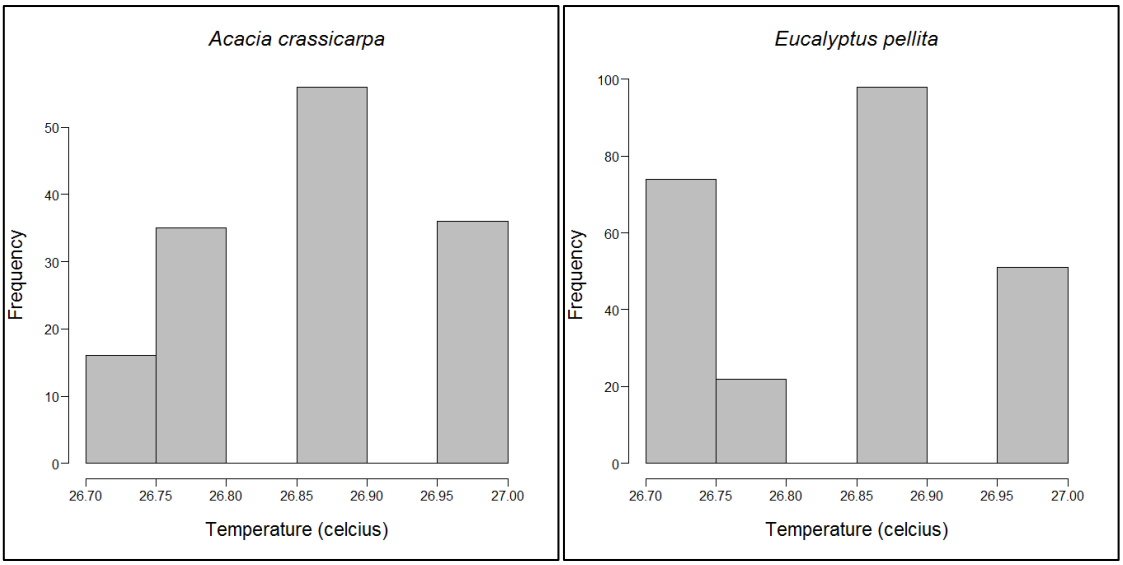

Figure 8 Mean annual temperature: *Acacia crassicarpa* (left) and *Eucalyptus pellita* (right)

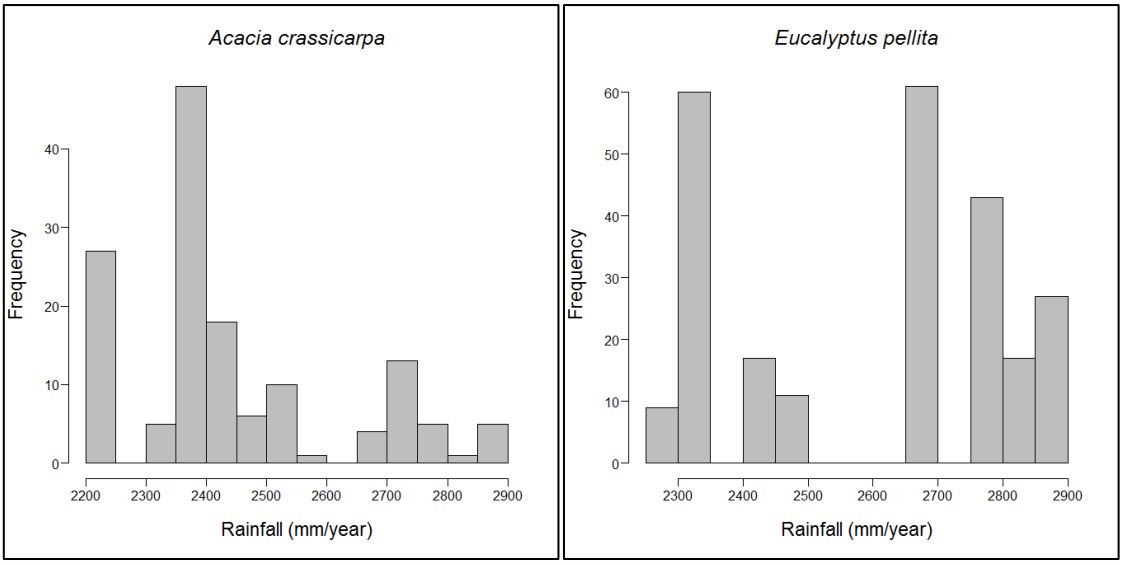

Figure 9 Mean precipitation: *Acacia crassicarpa* (left) and *Eucalyptus pellita* (right)

The number of PSPs used in this study varied among the species, and datasets for every species had limitations. For *Acacia mangium*, the PSPs were affected by bushfires that occurred from 2015 to 2016 in the study area, and so post-fire data were removed from this study. Other data limitations for this species were that singling (cutting the competitor stems on multileader trees) was also carried out in PSPs at age 4–6 months, the soil analysis had not been conducted in all PSPs and the company had not calculated the volume equation from their stem analysis data; hence, developing a volume equation for *Acacia mangium* became a part of the analysis during this study. While *Eucalyptus pellita* already had its own volume equation, a stem analysis had not been performed for *Acacia crassicarpa*, and so it was not possible to generate a volume equation. For this species, as an interim solution, we applied the volume equation for *Acacia mangium*, which had previously been planted in the same area.

Table 2 *The availability of PSP for this study*

Data available	*Acacia mangium*	*Acacia crassicarpa*	*Eucalyptus pellita*
Number of PSPs	503	374	358
PSP size (ha)	0.05	0.02	0.02
Shape of PSP	diamond	circular	circular
Singling	Yes	No	No
Thinning	No	No	No
Pruning	No	No	No
Number of PSP that only have one measurement	11	48	31
Number of PSPs that have an unusual increment	34	183	82
Number of PSPs affected by fire*	138	0	0
Number of PSP that can be used for this study	**320**	**143**	**245**

*Note: * after removing the post-fire data, these PSPs only have one measurement left.*

Table 3 Main variable of growth dataset for each species

Variable	*Acacia mangium* (320 plots)			*Acacia crassicarpa* (143 plots)			*Eucalyptus pellita* (245 plots)		
	Min	**Mean**	**Max**	**Min**	**Mean**	**Max**	**Min**	**Mean**	**Max**
Age (years)	0.90	2.50	6.12	0.84	2.54	6.00	0.96	2.48	5.25
MTH (m)	3.46	11.76	27.15	4.09	14.29	26.94	3.23	14.47	27.26
PMH (m)	3.10	12.03	26.28	4.35	15.07	27.40	3.30	15.02	27.05
G (m^2/ha)	1.13	9.91	29.59	0.88	11.58	29.07	0.50	9.88	26.36
N (stems/ha)	380	1,096	2,160	300	1,447	2,850	350	1,367	2,450
V (m^3/ha)	1.95	64.20	279.27	2.04	71.39	317.37	0.75	81.15	302.17
MTD (cm)	4.84	14.78	31.46	4.60	15.07	27.95	3.60	11.94	21.40
Mean dbhob (cm)	3.39	10.60	24.58	2.88	10.34	23.89	2.22	9.46	17.63
Max dbhob (cm)	5.00	16.22	42.60	4.70	15.72	33.90	3.70	12.37	21.70
Standard deviation of dbhob	0.74	2.72	7.10	0.93	3.00	7.32	0.47	2.00	5.46
Altitude (masl)	10.00	89.76	184.00	5.00	24.92	65.00	12.00	44.83	82.00
Annual mean temperature (°C)	25.50	25.83	26.20	26.70	26.88	27.00	26.70	26.84	27.00
Annual precipitation (mm)	2,394	2,466	2,569	2,237	2,442	2,852	2,288	2,585	2,894

Overall, these three species had a problem with unusual increments in their PSP datasets. Unusual increments included fluctuating increments of basal area or mean top height across rotation age (for instance, there were some decreasing basal area or mean top height (MTH) estimates in subsequent years) and an increases in the numbers of trees. Decreasing basal area or MTH might be related to mortality, but the cause of mortality was not recorded, and some decreases were associated with no change in numbers of trees in a plot. Based on observation in the field, windthrow of small intensity frequently occurred in the study area. Windthrow might lead to broken stems, broken tops, leaning trees or even fallen trees. If the dominant trees died in great numbers, either by self-thinning or catastrophic events, this may have led to decreases in the basal area or MTH. After consultation with data owners, growth intervals that had unusual or illogical increments were removed from the analysis. Detailed data that could be used in this study are shown in Table 2, and the general summaries of the main variables are shown in Table 3.

3.2. Preliminary Analysis

3.2.1. MTH Calculation

Mean top height (MTH) is a predicted value derived from the height and diameter equation by computing mean top diameter as an input variable. Mean top diameter (MTD) is defined as the

average diameter over bark at breast height (dbhob) of the 100 largest trees in 1 hectare. Owing to their plot sizes, the numbers of trees that were needed for calculating MTD was five trees for *Acacia mangium,* and two trees for *Acacia crassicarpa* and *Eucalyptus pelita*.

The height–dbhob model used in this study was the Näslund (Näslund, 1937) equation with the power value = −2 (Chapter IV) and a breast height that is used in Indonesia (1.3 m). Before generating the height–dbhob equations for growth dataset, the heights of trees that have defects, such as broken tops, broken stems and leaning trees, were excluded. But the dbhobs of these trees were kept for the MTD calculation.

The height–dbhob equation was specific for every PSP at every measurement; therefore there were 1,653 equations for *Acacia mangium,* 1,509 equations for *Acacia crassicarpa* and 1,326 equations for *Eucalyptus pellita.* These numbers depend on the number of PSPs and numbers of measurements of each PSP. However, the height–dbhob analysis could not be carried out for the plots that had fewer than three trees (called as less-trees PSPs). Hence, PMH (Predominant Mean Height) and MTH equations were created in order to calculate MTH values for less-trees PSPs. The number of trees in *Acacia crassicarpa* dataset were larger than three trees, so the PMH-MTH equation was not needed for this species. The Näslund equation is written in Equation 3.1, and PMH-MTH equation for *Acacia mangium* and *Eucalyptus pellita* are written in Equation 3.2 and 3.3 respectively.

$$MTH = 1.3 + \left(b + \frac{a}{dbhob}\right)^{-2} \quad \text{(Equation 3. 1)}$$

$$\text{MTH} = -0.316397 + 1.002930 * \text{PMH} \quad \text{for } \textit{Acacia mangium} \quad \text{(Equation 3. 2)}$$

$$\text{MTH} = -0.331180 + 0.988918 * \text{PMH} \quad \text{for } \textit{Eucalyptus pellita} \quad \text{(Equation 3. 3)}$$

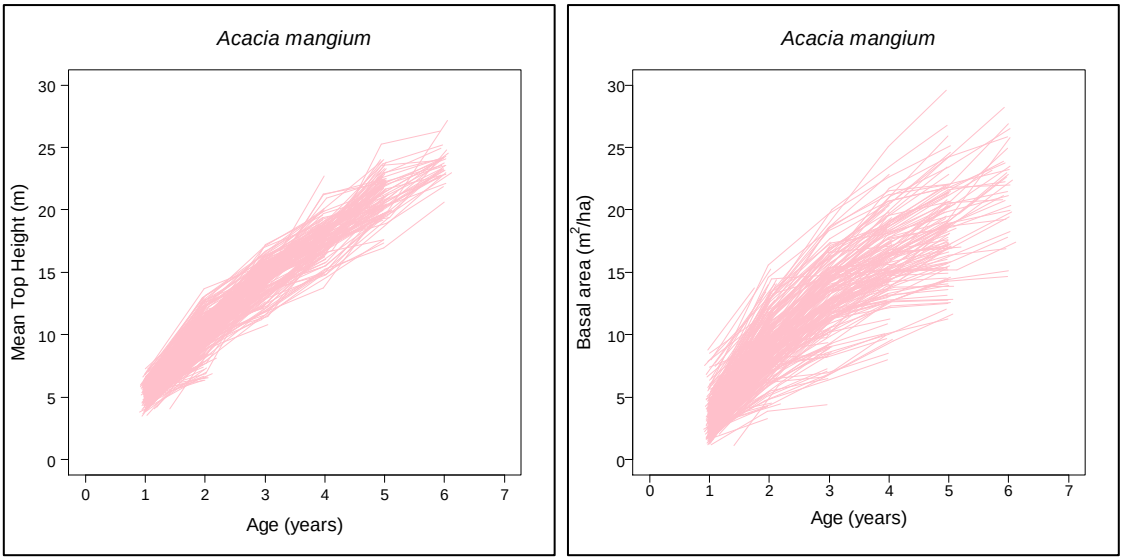

Figure 10 Growth trajectories of *Acacia mangium*: MTH (left) and basal area (right)

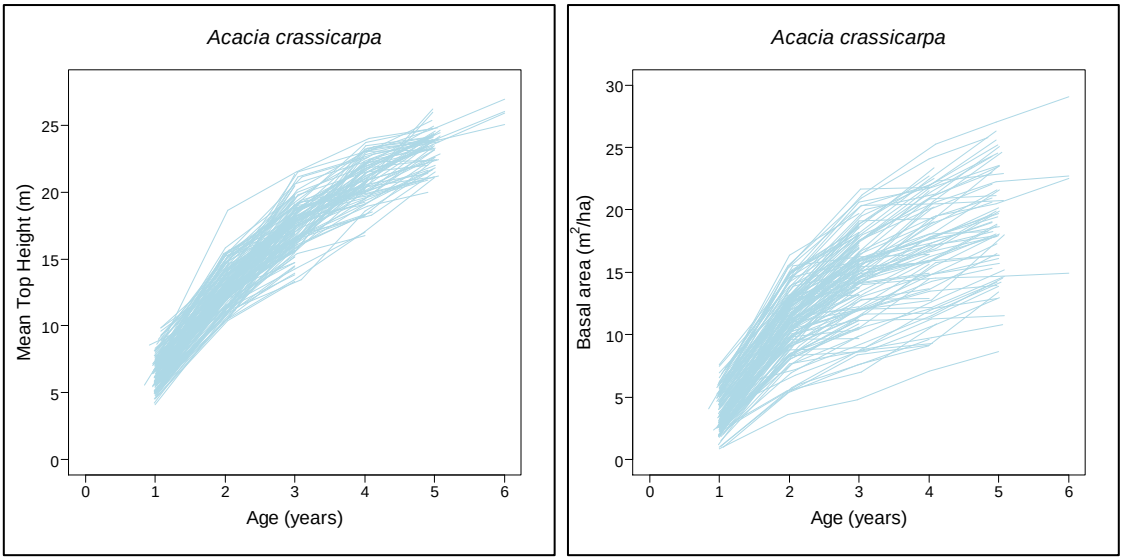

Figure 11 Growth trajectories of *Acacia crassicarpa*: MTH (left) and basal area (right)

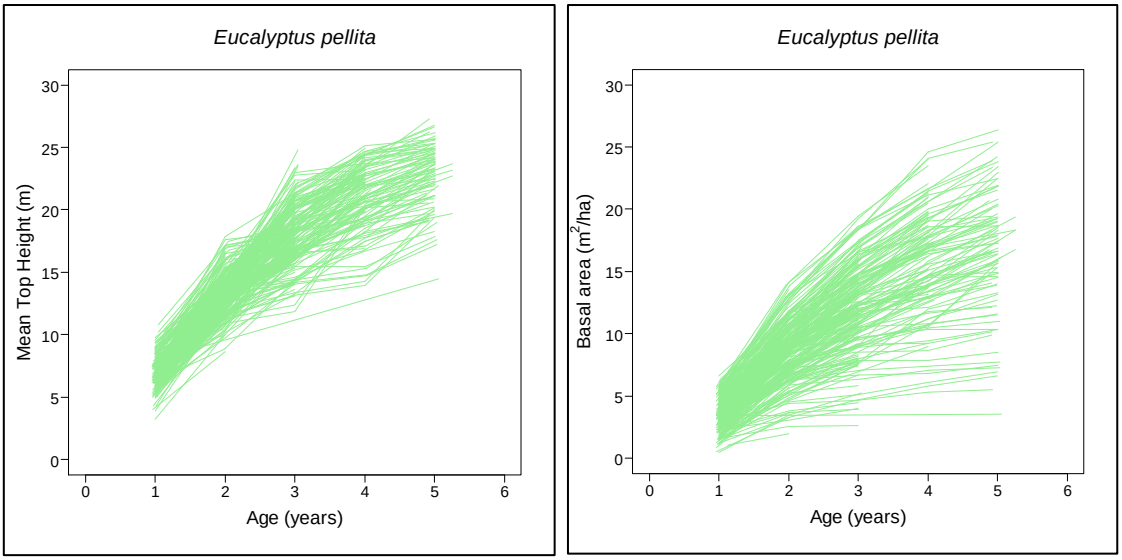

Figure 12 Growth trajectories of *Eucalyptus pellita*: MTH (left) and basal area (right)

3.2.2. Volume Equations

Calculating an individual-tree volume is needed before developing stand-volume equations. Of all species in this study, only *Eucalyptus pellita* had a tree volume equation. *Acacia mangium* still required the development of a volume equation from stem analysis data (Chapter IV). Sadly, no stem analysis or any research to generate volume equation had been done for *Acacia crassicarpa*. The AA company applied a volume equation for *Acacia mangium*, which had been previously planted in its area, for calculating the volume of *Acacia crassicarpa*. Since there were no relevant data for it, we accepted this equation for *Acacia crassicarpa*. The volume equations for these species were:

1. *Acacia mangium*

$$v = \left(0.27\left(-1.1070023 + 0.0055738\frac{h_t^{1.26}-1}{1.26} + 0.6162946\frac{\left(\left(\frac{dbhob}{100}\right)^2 h_t\right)^{0.24}-1}{0.24}\right) + 1\right)^{\frac{1}{0.27}} \quad \text{(Equation 3. 4)}$$

2. *Acacia crassicarpa* (derived from *A. mangium* equation)

$$v = \frac{exp^{-2.8144} dbhob^{1.6669} h_t^{1.1343}}{1000} \quad \text{for single stem} \quad \text{(Equation 3. 5)}$$

$$v = \frac{exp^{-4.9312} dbhob^{2.0166} h_t^{1.2493}}{1000} \quad \text{for double stems} \quad \text{(Equation 3. 6)}$$

$$v = \frac{exp^{-5.0422} dbhob^{1.8373} h_t^{1.3177}}{1000} \quad \text{for three stems} \quad \text{(Equation 3. 7)}$$

3. *Eucalyptus pellita*

$$v = \frac{dbhob^2 h_t}{23163.87 + 149.03\, dbhob} \quad \text{(Equation 3. 8)}$$

3.3. Growth Interval and Growth Equations

The growth interval for developing models of MTH, basal area (G), maximum dbhob (Dmax) and standard deviation of dbhob (Dstd) included all-possible intervals, but the growth intervals that were used for mortality (N) were all-possible intervals, longest intervals and one-year intervals. In the mortality section, models developed with these intervals were compared, to determine which interval to eventually use to develop mortality equation. In order to avoid pseudoreplication and also to prove the selected model, parameter checking was undertaken. The interval for parameter checking was just one interval for each plot that was randomly selected. We used nonlinear regression to construct all growth equations.

Table 4 *Summary of type data in stand-level datasets*

Stages	Methods	*Acacia mangium* (320 Plots)	*Eucalyptus pellita* (245 Plots)	*Acacia crassicarpa* (143 Plots)
Model construction	All-possible intervals	1207	917	636
	two-step regression (one-year interval)*	548	315	272
	Longest interval*	214	163	95
Validation (independent plots)	All-possible intervals	592	474	324
	two-step regression (one-year interval)*	273	152	134
	Longest interval*	106	82	48
Parameter checking		320	245	143
Comparing methods**		592	474	324

Note; *= *only using data with mortality occurring over a period (one-year interval).*
**= *only for comparing mortality approaches; data was same as validation dataset.*

Table 5 *Equations that were tested for growth, mortality and diameter distribution models*

Model No.	Author	Equation	*Acacia mangium*					*Acacia crassicarpa*					*Eucalyptus pellita*				
			MTH	**G**	**N**	**Dmax**	**Dstd**	**MTH**	**G**	**N**	**Dmax**	**Dstd**	**MTH**	**G**	**N**	**Dmax**	**Dstd**
Equation 1	Schumacher /Anamorphic I[1,6]	$Y_2 = e^{\ln(Y_1) - b\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 2	Lundqvist–Korf (Schumacher Anamorphic II)[1,8]	$Y_2 = Y_1 e^{\frac{-b}{T_2^c} - \frac{-b}{T_1^c}}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 3	Schumacher/ Polymorphic I[1,15]	$Y_2 = e^{\ln(Y_1)\left(\frac{T_1}{T_2}\right) + b\left(1 - \frac{T_1}{T_2}\right)}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 4	Schumacher/ Polymorphic II[1,15]	$Y_2 = e^{\ln(Y_1)\left(\frac{T_1}{T_2}\right)^c + b\left(1 - \left(\frac{T_1}{T_2}\right)^c\right)}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 5	Schumacher /Polymorphic III	$Y_2 = Y_1 \left(\frac{T_1}{T_2}\right) e^{b\left(1 - \frac{T_1}{T_2}\right)^c}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 6	Schumacher/ Polymorphic IV[1,3]	$Y_2 = e^{\ln(Y_1) - (b + cY_1)\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}$	√	√	√	√	√	√	√	X	√	√	√	√	X	√	√
Equation 7	Johnson–Schumacher[1]	$Y_2 = Y_1 e^{\frac{-b}{T_2 + c} - \frac{-b}{T_1 + c}}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 8	von Bertalanffy–Richards Anamorphic [2,13,15]	$Y_2 = Y_1 \left(\frac{1 - e^{-bT_2}}{1 - e^{-bT_1}}\right)^c$	√	√	X	√	√	√	√	X	√	√	√	√	√	√	√
Equation 9	von Bertalanffy – Richards Polymorphic I[2,15]	$Y_2 = b \left(\frac{Y_1}{b}\right)^{\frac{\ln(1 - e^{-cT_2})}{\ln(1 - e^{-cT_1})}}$	√	√	X	√	X	√	√	X	√	√	√	√	X	√	√
Equation 10	von Bertalanffy – Richards Polymorphic II[15]	$Y_2 = b \left\{ 1 - \left[1 - \left(\frac{Y_1}{b}\right)^{1-c} \right]^{\frac{T_2}{T_1}} \right\}^{\frac{1}{1-c}}$	√	X	X	√	√	X	X	X	√	X	X	X	X	√	X

Model No.	Author	Equation	*Acacia mangium*					*Acacia crassicarpa*					*Eucalyptus pellita*				
			MTH	**G**	**N**	**Dmax**	**Dstd**	**MTH**	**G**	**N**	**Dmax**	**Dstd**	**MTH**	**G**	**N**	**Dmax**	**Dstd**
Equation 11	Weibull[1,10,15]	$Y_2 = Y_1\left(\frac{1-e^{-bT_2^c}}{1-e^{-bT_1^c}}\right)$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 12	Weibull Polymorphic I[15]	$Y_2 = Y_1 e^{-b(T_2^c - T_1^c)} + d(1-e^{-b(T_2^c - T_1^c)})$	X	X	√	X	√	X	X	√	X	√	X	X	√	X	√
Equation 13	Monomolecular[1,4]	$Y_2 = Y_1\left(\frac{1-be^{-cT_2}}{1-be^{-cT_1}}\right)$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 14	Richards[1,4]	$Y_2 = Y_1\left(\frac{1-be^{-cT_2}}{1-be^{-cT_1}}\right)^{-\frac{1}{1-d}}$	X	√	X	X	X	X	√	X	X	X	X	√	X	X	X
Equation 15	Gompertz[1,4,9,15]	$Y_2 = Y_1\frac{e^{-be^{-cT_2}}}{e^{-be^{-cT_1}}}$	√	√	X	√	√	√	√	√	√	√	√	√	√	√	√
Equation 16	Gompertz Polymorphic I[15]	$Y_2 = e^{\ln(Y_1)e^{-b(T_2-T_1)}e^{c(1-b(T_2-T_1))}}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 17	Logistic I[1,4]	$Y_2 = Y_1\left(\frac{1+be^{-cT_1}}{1+be^{-cT_2}}\right)$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 18	Logistic II[3]	$Y_2 = Y_1\left(\frac{1-e^{-cT_2}}{1-e^{-cT_1}}\right)\left(\frac{1+e^{-b-cT_1}}{1+e^{-b-cT_2}}\right)$	√	√	X	X	X	√	√	X	X	X	√	√	X	X	X
Equation 19	Logistic III[3]	$Y_2 = Y_1\left(\frac{1-e^{-e^{cT_2}}}{1-e^{-e^{cT_1}}}\right)\left(\frac{1+e^{-b-e^{cT_1}}}{1+e^{-b-e^{cT_2}}}\right)$	√	√	X	√	√	√	√	X	√	√	√	√	X	√	√
Equation 20	Hossfeld Anamorphic[15]	$Y_2 = \frac{1}{\left(\left(\frac{1}{Y_1}\right) + b\left(\frac{1}{T_2^c} - \frac{1}{T_1^c}\right)\right)}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

Model No.	Author	Equation	*Acacia mangium*					*Acacia crassicarpa*					*Eucalyptus pellita*				
			MTH	**G**	**N**	**Dmax**	**Dstd**	**MTH**	**G**	**N**	**Dmax**	**Dstd**	**MTH**	**G**	**N**	**Dmax**	**Dstd**
Equation 21	Hossfeld Polymorphic[15]	$Y_2 = \frac{1}{\left(\left(\frac{1}{Y_1}\right)\left(\frac{T_2}{T_1}\right)^b + c\left(1-\left(\frac{T_2}{T_1}\right)^b\right)\right)}$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 22	Hossfeld IV[1,12]	$Y_2 = Y_1\left(\frac{T_2^c}{T_1^c}\right)\left(\frac{b+T_1^c}{b+T_2^c}\right)$	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√
Equation 23	Levakovic III[1,5,15]	$Y_2 = Y_1\left(\frac{T_2^2\,(b+T_1^2)}{T_1^2(b+T_2^2)}\right)^c$	√	√	X	√	√	√	√	X	√	√	√	√	X	√	√
Equation 24	Hyperbola I[1]	$Y_2 = Y_1\left(\frac{b+c^{\frac{1}{T_2}}}{b+c^{\frac{1}{T_1}}}\right)$	√	√	X	√	√	√	√	X	√	√	√	√	X	√	√
Equation 25	Hyperbola II	$Y_2 = Y_1\left(\frac{b+c^{\frac{1}{T_2}}+dT_2}{b+c^{\frac{1}{T_1}}+dT_1}\right)$	√	√	X	√	√	√	√	X	√	X	√	√	X	√	X
Equation 26	Anamorphic II*	$Y_2 = Y_1\left(\frac{T_2}{T_1}\right)^b e^{-b\left(T_2^c-T_1^c\right)}$	X	X	√	X	X	X	X	X	X	X	X	X	√	X	X
Equation 27	Anamorphic III*	$Y_2 = Y_1\left(\frac{T_2}{T_1}\right)^b$	X	X	√	X	X	X	X	√	X	X	X	X	√	X	X
Equation 28	Anamorphic IV*	$Y_2 = Y_1^{\left(\frac{T_2}{T_1}\right)^b}$	X	X	√	X	X	X	X	√	X	X	X	X	√	X	X
Equation 29	Polymorphic I*	$Y_2 = Y_1 e^{-b(T_2-T_1)} + c(1-e^{-b(T_2-T_1)})$	X	X	√	X	X	X	X	√	X	X	X	X	√	X	X
Equation 30	Exponential decay anamorphic*[5]	$Y_2 = Y_1 e^{-b(T_2-T_1)}$	X	X	√	X	X	X	X	√	X	X	X	X	√	X	X

Source: 1 =Burkhart and Tomé (2012), 2= Gadow and Hui (1999), 3= Kimberley and Ledgard (1998), 4=Seber and Wild (1989), 5=Casnati (2016), 6=Schumacher (1939), 7=Clutter (1963), 8=Lundqvist (1957), 9=Gompertz (1825), 10=Weibull (1951), 12=Peschel (1938), 13 =Pienaar and Turnbull (1973),14=Methol (2001)

*Note: * =additional equation for mortality, √= successfully tested, X = not successfully tested*

where: Y_2 = yield at time T_2, Y_1 = yield at time T_1, b, c and d = parameters, e = exp, ln = natural logarithm.

3.4. Model Selection

For validating models, datasets must be independent from datasets that are used for developing the models. Before creating models, a PSP dataset must be divided into two parts: one to create models and one for validating models (model benchmarking). There is no strict rule for how much data should be used for fitting a model and for validation. Most people used proportions of 4:1 (Lumbres et al., 2015; Zhao, 1999) or 3:1 (Krisnawati, Wang, & Ades, 2010; Návar, 2014; Putranto, 2017) to fit models and for validation.

Criteria used to choose the best model include lack of bias, having normal distributions of residuals and having small mean square error (MSE). However, since MSE squares all differences, this scale does not have the same scale as the original measurement (Walther, Moore, & Rahbek, 2005). Therefore, we dropped MSE as an index of precision for this study. Other statistics that are commonly used for test precision are SEE (Methol, 2001), MAE, RMSE, and MEF (Gadow & Hui, 1999). MEF was not used for this study as we explained in Chapter II. We used SEE instead of RMSE for precision test, since SEE and RMSE tend to have a similar result in a large dataset. Regardless of the sign of residuals, MAE provides us with essential information about the average of cumulative errors. Bias can be shown by mean residuals (MRES) to get a directional magnitude (overestimate or underestimate) of a model (Walther et al., 2005) and AIC estimates the performance of each model relative to other models.

In order to avoid blind judgement by only using those quantitative statistics, we also performed a qualitative analysis through graphs for model comparison. The advantages of graphs (Anscombe, 1973) are that they help us observe some broad features of data, to check whether the assumptions are reasonably correct and to investigate in what ways the assumptions are wrong (if they are wrong). The things that need to be looked for in graphical analysis are (Anscombe, 1973):

1) outliers (a few of the residuals much larger in magnitude than all others)
2) a curved regression of residuals on predicted values
3) progressive change in variability of residuals as the predicted values increase (check whether heteroscedasticity appears or not)
4) a skewed (or other abnormal) distribution of the residuals.

Based on all the considerations above, statistics that were used for this study were MRES, SEE, MAE and AIC. The smallest values of SEE, MAE and AIC are preferable. For MRES, the closest value to zero is desirable. The graph analyses for this study include residual plots that show the predicted value of the *x*-axis and residuals value in the *y*-axis to describe bias and histogram of residuals to see the normality of a model. In addition, we also used the Shapiro–Wilk test of normality. The graph analyses became a major consideration when we determined the chosen model. We used R software (R Core Team, 2013) for all data analyses.

CHAPTER IV

Preliminary Analysis

4.1. Compatible Taper and Volume Equation for *Acacia mangium*

4.1.1. Introduction

In a production forest, information on stem volume – either total volume or merchantable volume – is essential. Volume equations give information about the total volume of a tree from the diameter at breast height outside bark (dbhob) and the height of the tree. Merchantable volume, on the other hand, is not as simple to calculate as total volume. Usually, merchantable volume has a standard of upper stem diameter limitation and log length specification (Clutter, 1983). For instance, a pulp log of *Acacia mangium* usually requires a length of two metres with a small-end diameter of five centimetres. Sawlogs of *Instia bijuga* usually need to be four metres long with 55 cm minimum top diameter. It is not uncommon for a single tree to yield different products from each stem section (such as for plywood and pulp log) to maximise the profit.

The standard method to calculate merchantable volume in Indonesia is by fitting a regression model for a particular merchantable limit (Krisnawati, 2016). It is possible that the standard of merchantable logs might change due to market demand, the invention of a new technology in wood production, and many more. These changes could lead to many equations of merchantable volume just for a single species. Taper equations express an expected diameter at a certain height of a tree as a function of dbhob, total height and height above ground level (Clutter, 1983). By using taper equations, the volume for any given merchantable limit can be calculated.

The main issue in the volume calculation is that the summation of volume generated from a taper equation has a big difference from the volume that is produced from a volume equation. Demaerschalk (1972) first proposed compatible taper and volume equations to avoid these differences. Krisnawati (2016) later created a compatible taper and volume for *Acacia mangium* in South Sumatra, Indonesia. However, a single taper equation will not be sufficient to cover all stand conditions (Clutter, 1983).

The main objective of this study was to fit a volume equation for *Acacia mangium* in East Kalimantan as a preliminary analysis to generate stand volumes for a growth and yield model for this species. The additional objective was to create a compatible taper equation to obtain merchantable volume estimations of *Acacia mangium*.

4.1.2. Literature Review

4.1.2.1. Tree Volume

There have been many ways to determine the volume of a tree: the typical method of calculation is by using the formula of a geometric shape of a stem (cylinder, neioloid, paraboloid and cone). Other ways to determine the volume of a tree are the displacement method by using a xylometer, integration, graphical methods, height accumulation and bark volume (Husch et al., 2003). The geometric forms of different portions of a tree stem can be assumed to be a cylinder frustum for the stump; a frustum of a neiloid for the butt log; a frustum of a paraboloid for logs and upper logs; and the tip of a tree can be either a frustum of a cone or a paraboloid. The prominent formulas to determine a tree volume are Smalian, Huber's and Newton's formulas. Newton's formula is suitable for all frustums, but Huber's and Smalian's formulas are only suitable for frustums of paraboloids (Husch et al., 2003). However, Smalian's formula is ideal for a stem that is divided into short lengths (4 ft or about 1 metre) and it will give a good accuracy to calculate the merchantable volume (Husch et al., 2003).

Table 6 *Equations to compute cubic tree volume of important geometric shape (Husch et al., 2003)*

No	Geometric solid	Volume equation	Note
1	Cylinder	$v = A_b h$	
2	Paraboloid	$v = \frac{1}{2}(A_b h)$	
3	Cone	$v = \frac{1}{3}(A_b h)$	
4	Neiloid	$v = \frac{1}{4}(A_b h)$	
5	Paraboloid frustum	$v = \frac{h}{2}(A_b + A_u)$	Smalian's formula
		$v = A_m h$	Huber's formula
6	Cone frustum	$v = \frac{h}{3}\left(A_b + \sqrt{A_b A_u} + A_u\right)$	
7	Neiloid frustum	$v = \frac{h}{4}\left(A_b + + \sqrt[3]{A_b^2 A_u} + \sqrt[3]{A_u^2 A_b} + A_u\right)$	
8	Neiloid, cone or paraboloid frustum	$v = \frac{h}{6}(A_b + 4A_m + A_u)$	Newton's formula

Note: A_b = cross-sectional area at base, A_m = cross-sectional area at middle, A_u = cross-sectional area at upper end, h = height or length, v= volume

4.1.2.2. Compatible Taper Equations

Taper is defined as a reduction of stem diameter as height increases up the stem (Clutter, 1983). Taper functions provide information about the change of diameter up a tree stem, and thus estimates of log dimensions at a certain cut length (relative to height above the ground) can be obtained (Ounekham, 2009). As noted earlier, it is preferable to create a compatible taper equation that gives an identical result to that of a volume equation.

The theory of compatible taper equations is explained in an example below for the following notation, where:

h_t = total tree height (m)
dbhob =diameter over bark at breast height (cm)
h = distance up the stem from the ground (m)
v = volume from the tip of the tree to a point h_t-h with d diameter (m^3)
v_t = estimated total volume through a volume function (m^3)
d = diameter at point h (cm)
$z = (h_t-h)/h_t$, and $k = (\pi/4 \times 10^{-4})$

Given an example of a taper equation such as:

$$\frac{d^2}{dbhob^2} = \beta_1\left(\frac{h_t-h}{h_t}\right) + \beta_2\left(\frac{h_t-h}{h_t}\right)^2 + \cdots \beta_n\left(\frac{h_t-h}{h_t}\right)^n \quad \text{(Equation 4. 1)}$$

an example of a tree volume model is:

$$v_t = kdbhob^2 h_t \quad \text{(Equation 4. 2)}$$

and a volume function:

$$v = k\int_0^{h_t} d^2\, dh. \quad \text{(Equation 4. 3)}$$

The taper equation can be written as follows (substitute Equation 4.2 into Equation 4.1):

$$d^2 = \frac{v_t}{kh_t}\left(\beta_1\left(\frac{h_t-h}{h_t}\right) + \beta_2\left(\frac{h_t-h}{h_t}\right)^2 + \cdots + \beta_n\left(\frac{h_t-h}{h_t}\right)^n\right). \quad \text{(Equation 4. 4)}$$

If the k constant is removed from Equation 4.3 Then it follows:

$$v = \frac{v_t}{h_t}\int_0^{h_t} \beta_1\left(\frac{h_t-h}{h_t}\right) + \beta_2\left(\frac{h_t-h}{h_t}\right)^2 + \cdots + \beta_n\left(\frac{h_t-h}{h_t}\right)^n\, dh. \quad \text{(Equation 4. 5)}$$

Now, if $z = \left(\frac{h_t-h}{h_t}\right)$, then $dz = \frac{-1}{h_t\, dh}$ or $dh = -h_t\, dz$

(and in addition, $z = 0$ when $h = \mathrm{h_t}$ and $z = 1$ when $h = 0$), so

substituting z into Equation 4.5 and noting $\mathrm{h_t}$ cancels with the term outside the integral, it becomes

$$v = -v_t \int_1^0 \beta_1 z + \beta_2 z^2 + \cdots \beta_n z^n \, dz. \qquad \text{(Equation 4. 6)}$$

Integrating gives: $v = -v_t \, {}_1^0\left[\beta_1 \frac{z^2}{2} + \beta_2 \frac{z^3}{3} + \cdots + \beta_2 \frac{z^{n+1}}{n+1}\right]$

and evaluating: $v = -v_t \left(\frac{\beta_1}{2} + \frac{\beta_2}{3} + \cdots + \frac{\beta_n}{n+1}\right)$.

Hence, this provides: $\left(\frac{\beta_1}{2} + \frac{\beta_2}{3} + \cdots + \frac{\beta_n}{n+1}\right) = 1$.

It follows that the estimated volume from a taper equation is identical to that from a volume equation. To produce fully compatible taper equations, we need to use a restriction of parameters $\left(\left(\frac{\beta_1}{2} + \frac{\beta_2}{3} + \cdots + \frac{\beta_n}{n+1}\right) = 1\right)$.

4.1.2.3. Variance Heterogeneity

Assumptions of the analysis of linear models, as demonstrated by analysis of variance and multiple regression, are simplicity of error structure $(E[y])$, constancy of error variance, independence of observations and normality of residual distribution (Box & Cox, 1964). However, these assumptions are difficult to satisfy in biological applications. Heterogeneity of variance commonly happens when larger objects give more variation (for instance, larger trees have more variation between them than smaller trees do). Techniques for solving the heterogeneity of variance are either applying a weighted least-squares method or transformation.

One of the transformations is a scaled power transformation (Cook & Weisberg, 1999), also known as a Box-Cox transformation (Seber & Wild, 1989) which is written:

$$y^{(\lambda)} = \begin{cases} \dfrac{y^\lambda - 1}{\lambda}, & \lambda \neq 0 \\ \log y, & \lambda = 0. \end{cases}$$

For a model $y \approx f(x, \theta)$, the power transform on both sides (Leech, 1975) was used (Equation 4.7).

$$y_i^{(\lambda_1)} = f\ (x_i, \theta)^{(\lambda_1)} + \varepsilon_i, \qquad \text{(Equation 4. 7)}$$

where the ε_i are independent and identically distributed $N(0, \sigma^2)$.
and the power-transformed weight least squares method (Seber & Wild, 1989):

$$y_i \approx f\ (x_i, \theta), \qquad \text{(Equation 4. 8)}$$

where values of ε_i are independent of $N\left(0, \sigma^2 f\ (x_i, \theta)^{(\lambda_2)}\right)$ and $\lambda_2 \approx 2 - 2\ \lambda_1$.

The weighted least squares model in Equation 4.8 depends on the estimated parameter, and this equation is preferred for a linear function. On the other hand, Equation 4.7 is more suitable for nonlinear functions. The distribution of Equation 4.7 gives a homogeneity of variance due to transformation and, in contrast, the weighted least squares methods still provide a heterogeneity of variance due to the unchanged *y* distribution (Seber & Wild, 1989).

Weighted factors commonly used in the volume equation are $\frac{1}{dbhob^2}$ and $\frac{1}{dbhob^2 h_t}$. Schreuder and Williams (1998) found that there was no significant difference between these two factors for felled trees, but there was a significant difference for standing trees. In addition, Meng and Tsai (1986) suggested using $\frac{1}{dbhob^2}$ instead of $\frac{1}{dbhob^4}$.

4.1.3. Method

4.1.3.1. Volume

Data for this study were obtained from stem analysis of 192 sample trees in the FSS area. These trees were selected purposely to represent age ranges from 1 to 6 years with 32 trees of each age. Dbhob ranged from 5.8 to 28.1 cm and total height varied from 4.79 to 32.18 metres (Table 7). The length of each section was mostly 100 cm but for small trees the sections were sometimes a 50-cm length. Diameter and bark thickness were measured for every section up to 1-cm small-end diameter (SED). The merchantable diameter limit used in this company was 5 cm. The last section is non-merchantable and it varied to make the length from 5-cm large-end diameter (LED) to the 1-cm SED. The bark thickness was measured once for every cross-section of log. Hence, it was multiplied by two in order to get the diameter under bark. The volume of each section was calculated using Smalian's formula and the total volume was derived from the summation of each section.

Table 7 *Volume and taper dataset of* Acacia mangium

Variable	Unit	Volume dataset		Taper dataset	
		Model fitting	Validation	Model fitting	Validation
Number of trees		144	48		
Number of measurements				580	192
Age					
Minimum	years	1.00	1.00	1.00	1.00
Mean		3.50	3.50	4.15	4.12
Maximum		6.00	6.00	6.00	6.00
Diameter over bark at breast height (dbhob)					
Minimum	cm	6.10	5.80	5.80	5.80
Mean		15.73	15.82	17.79	17.72
Maximum		28.10	27.90	28.10	28.10
Standard deviation		5.97	6.19	5.72	5.66
Total Height (h_t)					
Minimum	m	4.85	4.79	4.79	4.79
Mean		17.71	17.83	20.63	20.48
Maximum		29.80	32.18	32.18	32.18
Standard deviation		7.52	7.91	6.62	6.91
Diameter over bark (do)					
Minimum	cm			1.00	1.00
Mean				11.86	11.47
Maximum				26.90	23.90
Standard deviation				5.68	5.30
Diameter under bark (du)					
Minimum	cm			0.94	0.94
Mean				11.80	11.41
Maximum				26.68	23.80
Standard deviation				5.66	5.29
Total volume over bark (vob)					
Minimum	m^3	0.01090	0.00900		
Mean		0.23250	0.23471		
Maximum		0.74520	0.78180		
Standard deviation		0.20579	0.20758		
Total volume under bark (vub)					
Minimum	m^3	0.01070	0.00880		
Mean		0.23030	0.23250		
Maximum		0.73930	0.77550		
Standard deviation		0.20424	0.20609		

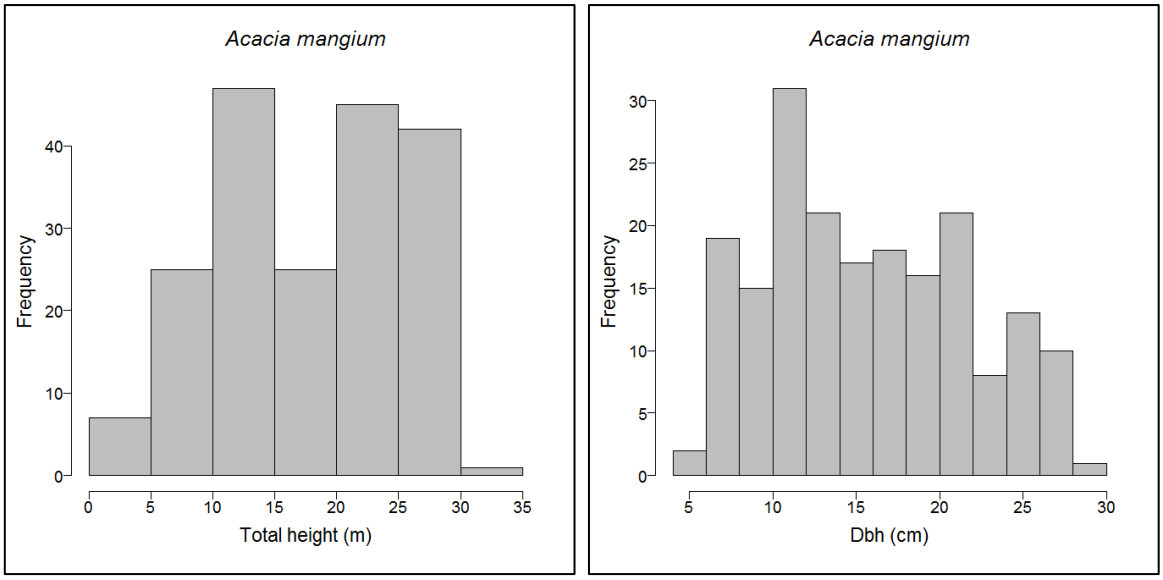

Figure 13 The distribution of total height (left) and diameter at breast height (cm) from stem analysis data.

Volume equations commonly used for estimation of individual tree volumes are constant form variable, combined variable, generalised combined variable, ogarithmic, generalised logarithmic and Honer transformed variable (Clutter, 1983; Husch et al., 2003). Trees were randomly selected from 75 % of total data for model development, with the number of trees being 21 trees per age class. The rest were used for independent validation.

Table 8 *Equation form commonly used for estimation of individual tree volumes*.

No	Name	Equation form	Equation number
1	Constant form factor (CFF)	$v = \beta_0\, dbhob^2 h_t$	Equation 4. 9
2	Combined variable (CbV)	$v = \beta_0 + \beta_1\, dbhob^2 h_t$	Equation 4. 10
3	Generalised combined variable (GCV)	$v = \beta_0 + \beta_1\, dbhob^2 + \beta_2\, h_t + \beta_3\, dbhob^2 h_t$	Equation 4. 11
4	Logarithmic (Log)	$v = \beta_0\, dbhob^{\beta_1} {h_t}^{\beta_2}$	Equation 4. 12
5	Generalised Logarithmic (GLog)	$v = \beta_0 + \beta_1\, dbhob^{\beta_2} {h_t}^{\beta_3}$	Equation 4. 13
6	Honer transformed variable (HTV)	$v = \frac{dbhob^2}{\beta_0 + \beta_1\, {h_t}^{-1}}$	Equation 4. 14

Note : v= volume (m^3), dbhob=diameter over bark at breast height (cm), h_t = total height (m), $dbhob^2$ = quadratic diameter over bark at breast height (m^2), $dbhob^2h_t$ = $dbhob^2$ x total height (m^3)

From Figure 14 below, it seems that there were more variations in larger values of $dbhob^2h_t$ and volume than in the smaller values. To avoid this heterogeneity, weighted least squares methods or transformations were conducted. Weighted least squares fitting was used for all

equations (Equations 4.9–4.14) and transformations were only carried out for Equations 4.10–4.13 (only four models). The log transformation was used for Equation 4.12 and scaled power transformations were used for the three remaining models.

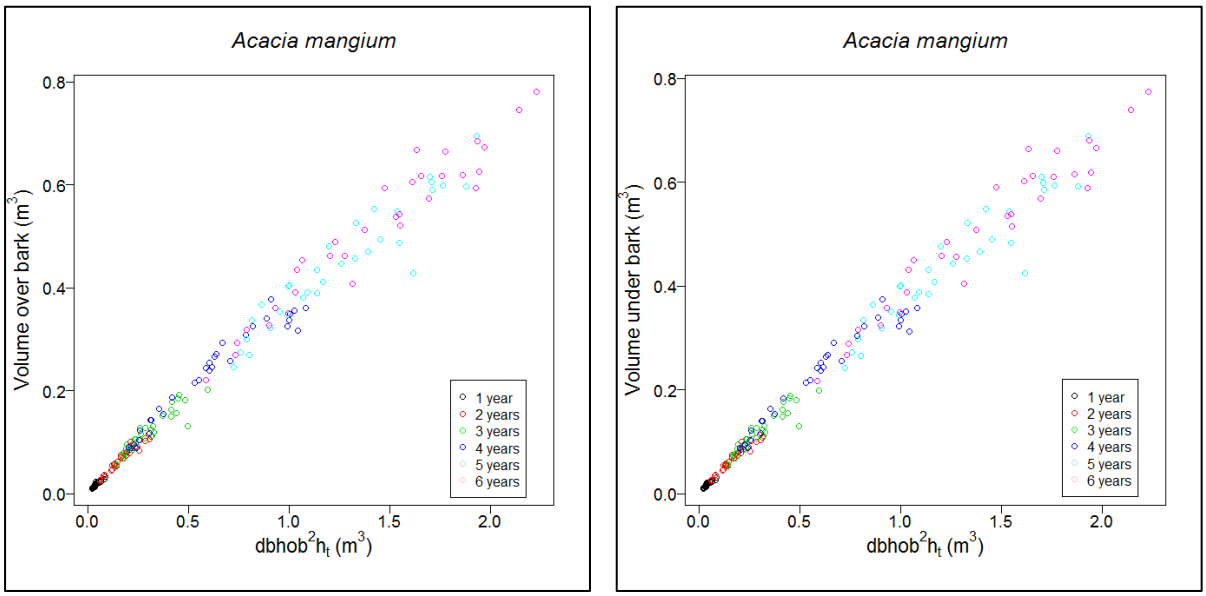

Figure 14 Volume over bark (left) and volume under bark (right) against $dbhob^2h_t$ in each age range

The log transformation of Equation 4.12 gives:

$$\log(v) = \log(\beta_0) + \beta_1 \log(dbhob) + \beta_2 \log(h_t)$$

The scaled power transformations (SPT) give:

$$\left(\frac{v^{\lambda v}-1}{\lambda_v}\right) = \beta_0 + \beta_1 \left(\frac{dbhob^{2\,h_t{}^{\lambda d^2 h}}-1}{\lambda_{d^2h}}\right) \quad \text{(Equation 4.11)}$$

$$\left(\frac{v^{\lambda v}-1}{\lambda_v}\right) = \beta_0 + \beta_1 \left(\frac{dbhob^{2\lambda d^2}-1}{\lambda_{d^2}}\right) + \beta_2 \left(\frac{h_t{}^{\lambda h}-1}{\lambda_h}\right) + \beta_3 \left(\frac{dbhob^{2\,h_t{}^{\lambda d^2 h}}-1}{\lambda_{d^2h}}\right) \quad \text{(Equation 4. 13)}$$

$$\left(\frac{v^{\lambda v}-1}{\lambda_v}\right) = \beta_0 + \beta_1 \left(\frac{dbhob^{\lambda d}-1}{\lambda_d}\right)^{\beta_2} + \left(\frac{h_t{}^{\lambda h}-1}{\lambda_h}\right)^{\beta_3} \quad \text{(Equation 4.14)}$$

where:

$\lambda_V = lambda\ value\ of\ v$, $\lambda_d = lambda\ value\ of\ dbhob$, $\lambda_h = lambda\ value\ of\ h_t$,

$\lambda_{d^2} = lambda\ value\ of\ dbhob^2$, $\lambda_{d^2h} = lambda\ value\ of\ dbhob^2h_t$

4.1.3.2. Taper

There were 3087 sectional data of height and diameter pairs from 192 sample trees. These enormous data were randomly reduced to 25% in order to get independent datasets. It gave 580 data for model construction and 192 data for model validation (Table 7). The single taper

functions and the compatible taper equations were tested in this study. The compatible taper functions used in this study were derived from polynomial functions (Goulding & Murray, 1976).

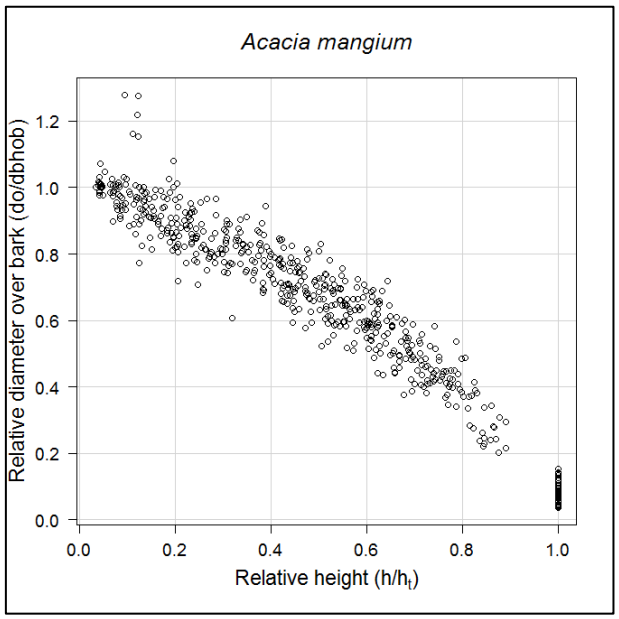

Figure 15 Relative height against relative diameter over bark of 580 taper data

Table 9 *Single taper functions*

No	Name	Equations	Equation number
1	Ormerod (1973)	$d = \beta_1 dbhob \left(\frac{h_t - h}{h_t - 1.3}\right)^{\beta_2}$	Equation 4. 15
2	Biging (1984)	$d = dbhob\left[\beta_1 + \beta_2 log\left(1 - \frac{h^{\frac{1}{\beta_3}}}{h_t}\left(1 - e^{\frac{-\beta_1}{\beta_2}}\right)\right)\right]$	Equation 4. 16
3	General taper	$\frac{d}{dbhob} = \beta_0 + \beta_1 \frac{h}{h_t} + \beta_2 \left(\frac{h}{h_t}\right)^2 + \beta_3 \left(\frac{h}{h_t}\right)^3$	Equation 4. 17
4	Kozak, Munro, and Smith (1969)	$d^2 = dbhob^2\left(\beta_1\left(\frac{h}{h_t} - 1\right) + \beta_2\left(\frac{h^2}{h_t{}^2} - 1\right)\right)$	Equation 4. 18
5	Sharma; and Oderwald (2001)	$d^2 = dbhob^2 \left(\frac{h}{1.3}\right)^{2-\beta_1}\left(\frac{h_t - h}{h_t - 1.3}\right)$	Equation 4. 19

Note: dbhob=diameter over bark at breast height (cm), h_t = total height (m), $dbhob^2$ = quadratic diameter over bark at breast height (cm^2), h = distance up from the stem from the ground (m), d=diameter at h point (cm).

Table 10 *Compatible taper equations tested in this study*

No	Equations	Equation number
1	$d^2 = \frac{v_t}{k\,h_t}\left(2\beta_1 z^1 + 3\beta_2 z^2 + 4\beta_3 z^3\right)$	Equation 4. 20
2	$d^2 = \frac{v_t}{k\,h_t}\left(2\beta_1 z^1 + 3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4\right)$	Equation 4. 21
3	$d^2 = \frac{v_t}{k\,h_t}\left(2\beta_1 z^1 + 3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5\right)$	Equation 4. 22
4	$d^2 = \frac{v_t}{k\,h_t}\left(2\beta_1 z^1 + 3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5 + 7\beta_6 z^6\right)$	Equation 4. 23
5	$d^2 = \frac{v_t}{k\,h_t}\left(3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4\right)$	Equation 4. 24
6	$d^2 = \frac{v_t}{k\,h_t}\left(3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5\right)$	Equation 4. 25
7	$d^2 = \frac{v_t}{k\,h_t}\left(3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5 + 7\beta_6 z^6\right)$	Equation 4. 26
8	$d^2 = \frac{v_t}{k\,h_t}\left(4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5\right)$	Equation 4. 27
9	$d^2 = \frac{v_t}{k\,h_t}\left(4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5 + 7\beta_6 z^6\right)$	Equation 4. 28
10	$d^2 = \frac{v_t}{k\,h_t}\left(4\beta_3 z^3 + 5\beta_4 z^4 + 6\beta_5 z^5 + 7\beta_6 z^6 + 8\beta_7 z^7\right)$	Equation 4. 29

Note: d^2 = quadratic diameter at point h (cm^2), v_t = estimated total volume through a volume function (m^3), h_t = total height (m), h= distance up the stem from the ground (m), $z = (h_t-h)/h_t$, and $k = (\pi/4 \times 10^{-4})$

4.1.3.3. Model Evaluation

All data for generating volume and taper equations were divided into two: to create the models and for validating the models. Of 192 trees, 144 trees were used in model fitting and the remaining trees were used for validation for generating the volume equation. Meanwhile, due to the large sectioning dataset of the taper dataset from these 192 trees, we firstly reduced the number of data to 25% (772 data). The taper dataset was randomly selected, hence some of the trees were used for validation and model fitting. Although the trees may be the same, the section of the trees that were used in model fitting were independent of the section that were used in validation.

The statistics that were used in this study were mean residuals (MRES), standard error of estimates (SEE), and mean absolute error (MAE). We used RMSE instead of SEE, since the number of parameters in the taper equation was not considered for the validation dataset (Methol, 2001). The choice of best model was based on the rank of the statistical values mentioned above, where the smallest values are the most desirable. However, residual plots and a histogram of normality became a major consideration for this study. For data analysis, we used R software (R Core Team, 2013) both in volume and taper modelling.

Standard error of estimates (SEE) $= \sqrt{\frac{\Sigma\,(Y_i - \hat{Y}_i)^2}{n-p}}$

Mean residuals (MRES) = $\frac{\Sigma(Y_i - \hat{Y}_i)}{n}$

Mean absolute error (MAE) = $\frac{\Sigma|Y_i - \hat{Y}_i|}{n}$

Standard error of estimates (SEE) validation= RMSE $= \sqrt{\frac{\Sigma\,(Y_i - \hat{Y}_i)^2}{n}}$

where:

n = observations, p = parameters,

Y_i = actual value of Y for observation i, $\hat{Y}_i$ = predicted value of Y for observation i,

4.1.4. Results and Discussion

4.1.4.1. Volume

From the six equations tested, Equation 4.11 and Equation 4.14 produced insignificant results in parameter β_0. The $dbhob^2$ parameter was removed from Equation 4.11 to get all significant parameters. Equation 4.13 had the smallest MAE both in model construction and validation. Equations 4.10 and 4.11 presented a small bias (small MRES) compared with others, although these models tended to slightly overestimate in model fitting. Based on SEE values, Equations 4.12 and 4.13 were more precise than the others. Overall, the results of these six equations exhibited a heteroscedasticity of residuals. The histogram of Equation 4.11 was closer to a normal distribution than the others were (Figure 17).

The heteroscedasticity pattern violated the assumption of a constant variance in error. Hence, either a weighted least-squares method or a transformation was needed prior to carrying out a regression. Weighted least-squares methods were used for these equations using $1/dbhob^2h_t$ as a weight factor for Equations 4.10 and 4.11 and $1/dbhob^2$ as a weight factor for Equations 4.12

and 4.13. A scaled power transformation was used for Equations 4.10, 4.11 and 4.13, and a log transformation was used for Equation 4.12. The result of the weighted least-squares and the transformed models are shown in Tables 12 and 13.

Table 11 *Statistical values of model fitting and validation of volume over bark*

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.9	0.02622 (5)	6.82 x 10^{-03} (6)	0.01879 (6)	0.0349999 (6)	0.00903 (6)	0.20348 (6)
4.10	0.02432 (4)	-3.08 x 10^{-18} (1)	0.01717 (4)	0.03279527 (4)	0.00221 (1)	0.19923 (2)
4.11#	0.02095 (3)	-6.00 x 10^{-18} (2)	0.01448 (3)	0.02889176 (3)	0.00221 (1)	0.19941 (3)
4.12	0.01873 (2)	1.04 x 10^{-03} (4)	0.01263 (2)	0.02764139 (1)	0.00325 (4)	0.20011 (4)
4.13	0.01856 (1)	8.62 x 10^{-10} (3)	0.0125 (1)	0.02781977 (2)	0.00221 (3)	0.19904 (1)
4.14*	0.02626 (6)	5.97 x 10^{-03} (5)	0.01828 (5)	0.03498532 (5)	0.00818 (5)	0.20302 (5)

*Note: #=after removing dbhob^2, *= some parameters in models were insignificant.*
Ranking is shown in brackets (The smallest rank was preferable).

Table 12 *Statistical value of weighted least-squares method of volume over bark*

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.9	0.02999 (3)	-7.75 x 10^{-18} (2)	0.01911 (6)	3.875559 (6)	0.00221 (1)	0.20682 (6)
4.10	0.02698 (2)	-3.97 x 10^{-17} (3)	0.01713 (4)	3.59375 (4)	0.00221 (1)	0.20372 (4)
4.11#	0.02312 (1)	5.13 x 10^{-18} (1)	0.01412 (3)	3.056957 (3)	0.00221 (1)	0.19996 (3)
4.12	0.09535 (5)	2.15 x 10^{-04} (6)	0.01251 (2)	2.756699 (1)	0.00242 (6)	0.19972 (2)
4.13	0.09477 (4)	1.23 x 10^{-04} (4)	0.01225 (1)	2.761009 (2)	0.00233 (4)	0.19952 (1)
4.14*	0.1312 (6)	1.83 x 10^{-04} (5)	0.01781 (5)	3.716549 (5)	0.00239 (5)	0.20398 (5)

*Note: #=after removing dbhob^2, *= some parameters in models were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

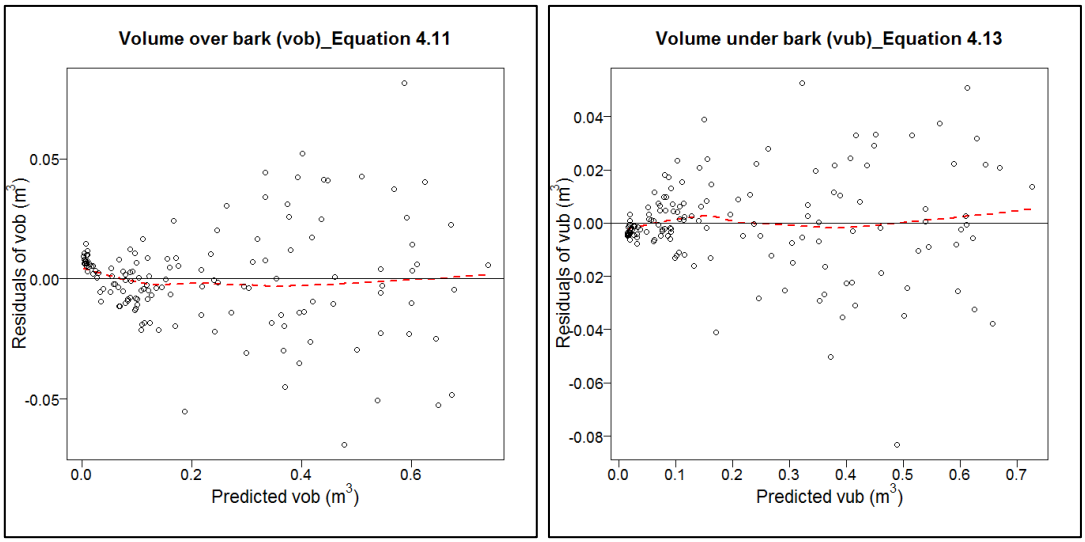

Figure 16 Examples of heteroscedasticity: Equation 4.11 (left) and Equation 4.13 (right)

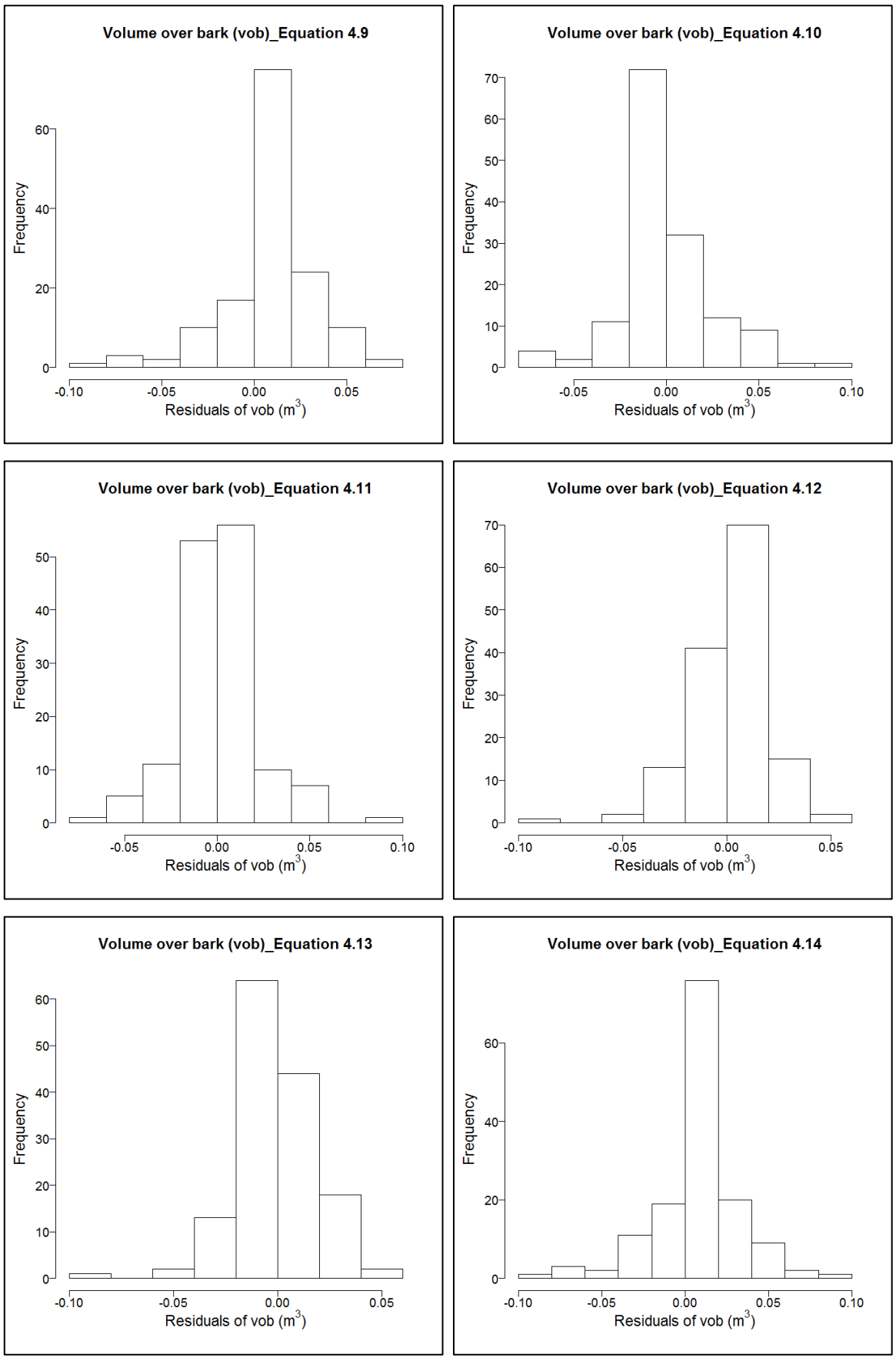

Figure 17 Histogram of residuals in model fitting of volume over bark (vob)

Table 13 *Statistic value of transformation of four best model of volume over bark*

Equation	Model fitting**			Validation**		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.10	0.02259 (4)	4.99 x 10^{-04} (2)	0.01455 (4)	0.2627329 (4)	0.00668 (2)	0.20631 (4)
4.11#	0.01884 (2)	6.22 x 10^{-04} (3)	0.01229 (2)	0.2607129 (1)	0.00716 (3)	0.20542 (2)
4.12	0.02042 (3)	1.66 x 10^{-03} (4)	0.01313 (3)	0.2616492 (3)	0.00564 (1)	0.20483 (1)
4.13	0.01875 (1)	4.08 x 10^{-04} (1)	0.01212 (1)	0.2614329 (2)	0.00738 (4)	0.20571 (3)

*Note: #=after removing dbhob^2, **= the value after back transform.*
Ranking is shown in brackets. (The smallest rank was preferable).

Overall, the weighted least-squares equations (Table 12) had smaller MRES values than the transformed equations (Table 13). However, standard errors (SEEs) of the weighted least-squares equations were larger than for the transformed equations (after back-transformation). Moreover, residual plots of the weighted least-squares method still demonstrated a heteroscedastic pattern (Figure 18). Therefore, the transformed models were chosen in preference to the weighted least-squares models.

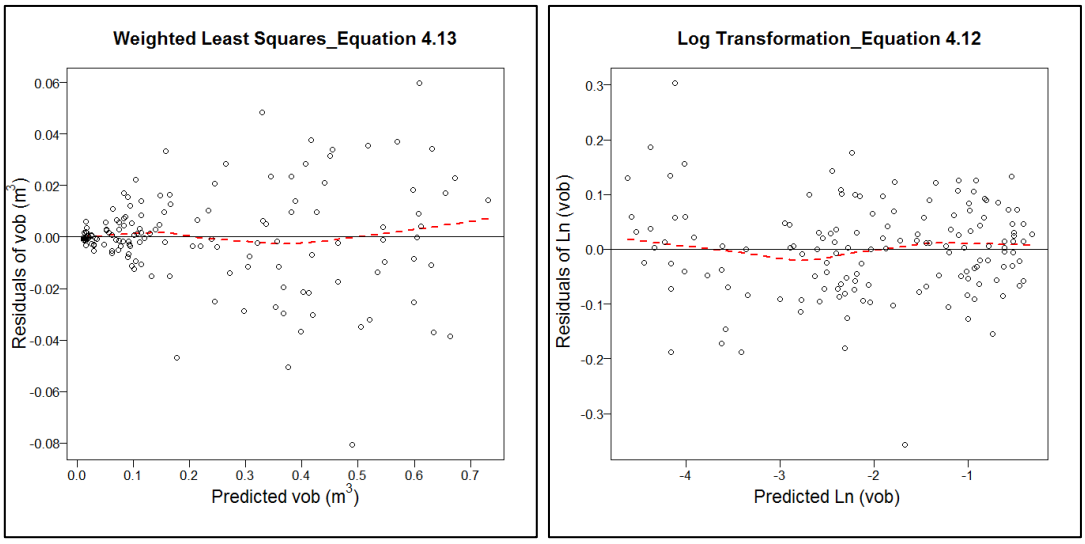

Figure 18 Weighted least squares of Equation 4.13 (left) and logarithmic transformation of Equation 4.12 (right)

Based on validation (Table 13), the best model to predict the volume of *Acacia mangium* was Equation 4.12 with a log transformation, followed by Equations 4.11 and 4.13. However, Figure 18 shows the residual plots of Equation 4.12 look like the inverse of heteroscedasticity (smaller value of predicted vob had more variation in residuals than its larger values). The scaled power transformations (SPT) of Equations 4.11 and 4.13 (Figure 19) seemed to have a constant variance.

Based on Table 13, the scaled power transform of Equation 4.13 fitted the best in model fitting compared with Equation 4.11; however, the difference between them was very small.

Validation of Equation 4.11 gave a better result by having smaller RMSE, MRES and MAE values compared with Equation 4.13. Considering the simplicity of the model, Equation 4.13 is more complicated than Equation 4.11. Therefore, we chose Equation 4.11 (scaled power transformation of general combined variable after removing the dbhob^2 parameter).

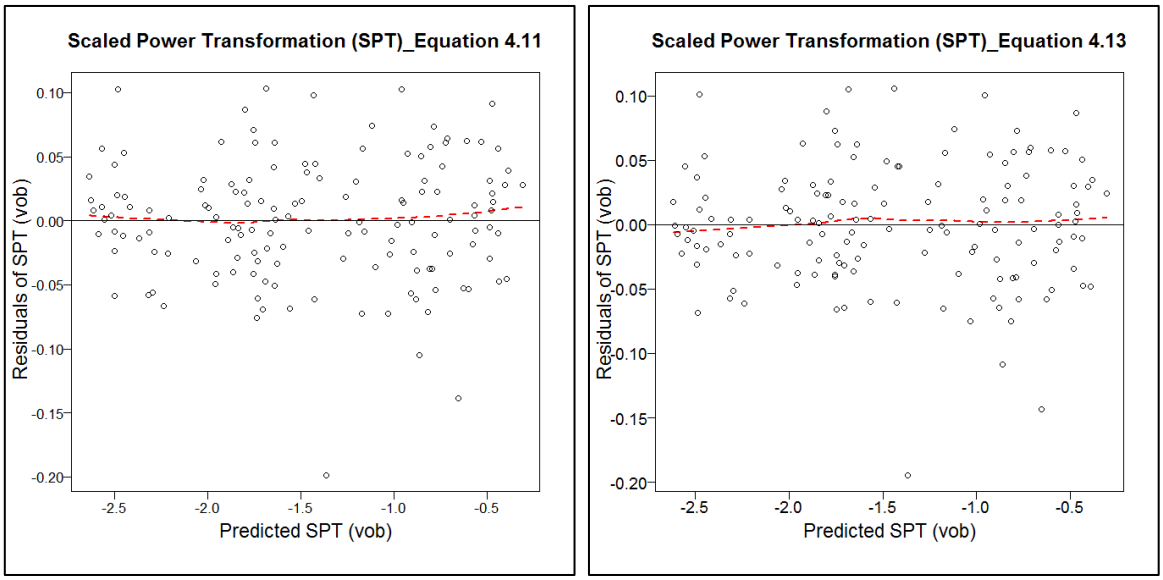

Figure 19 Scaled power transformation of Equation 4.11 (left) and Equation 4.13 (right) in model construction

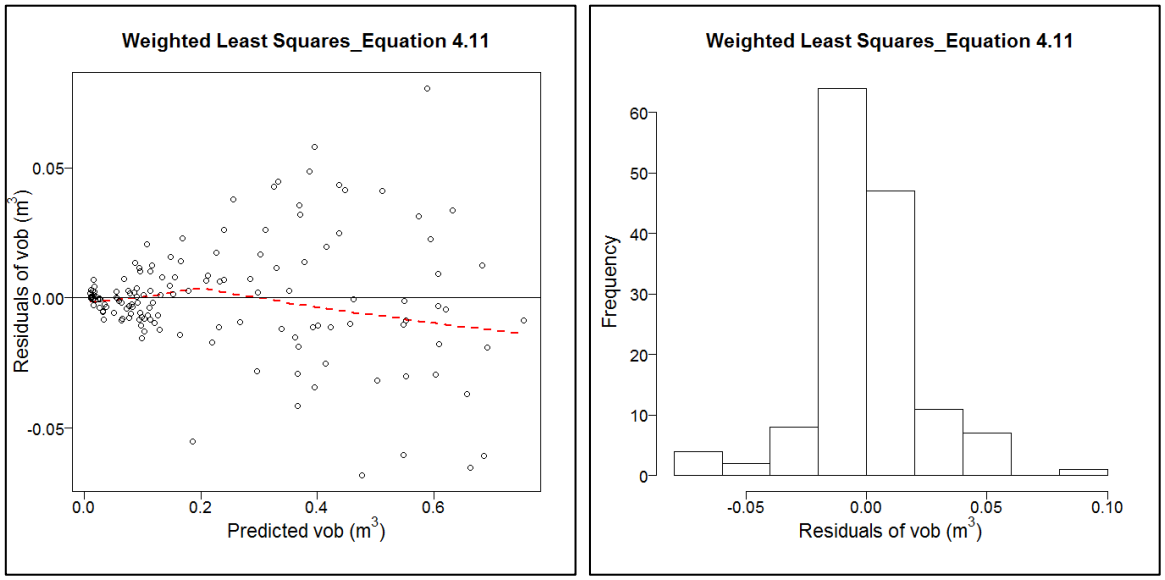

Figure 20 Weighted least-squares method of Equation 4.11 in model construction

The comparison of residuals of the weighted least-squares method and the back transformation of Equation 4.11 can be seen in Figures 20 and 21. The residual plots of the weighted least-squares method seem similar to the original equation in Figure 16 and still show heteroscedasticity, and hence the assumption of error term is still violated. Using a scale power transformation prior regression analysis gave a fulfilment of error assumption about constant

variance. The result of this equation after a back transform (Figure 21) produced less biased residuals (see the predicted vob from 0-0.4 m^3) and normality compared with the weighted least-squares method in Figure 20. The back-transformed equation also produced a better statistic outcome than its original equation.

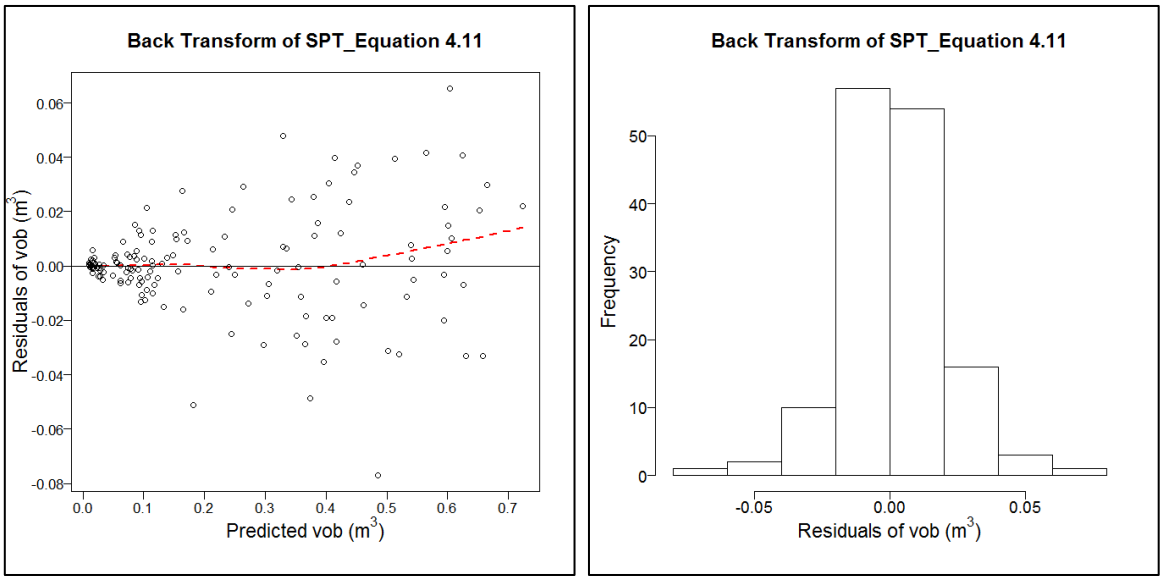

Figure 21 Back transform of scale power transformation (SPT) of Equation 4.11 in model construction

In conclusion, the chosen model for calculating volume over bark (vob) of *Acacia mangium* was the general combined variable (GCV) with scaled power transformation. It was also applied for volume under bark (vub), and gave the same result. The information on model construction and validation of volume under bark (vub) is given in Appendix A. The best equation is written below:

$$vob = (\lambda_v \left(a_0 + a_1 \frac{{h_t}^{\lambda_h} - 1}{\lambda_h} + a_2 \frac{\left(dbhob^2 h_t\right)^{\lambda_{d^2h}} - 1}{\lambda_{d^2h}}\right) + 1)^{\frac{1}{\lambda_v}}$$

$$vub = (\lambda_v \left(b_0 + b_1 \frac{{h_t}^{\lambda_h} - 1}{\lambda_h} + b_2 \frac{\left(dbhob^2 h_t\right)^{\lambda_{d^2h}} - 1}{\lambda_{d^2h}}\right) + 1)^{\frac{1}{\lambda_v}}$$

with:

$\lambda_v = 0.27$, $\lambda_h = 1.26$, $\lambda_{d^2h} = 0.24$, $a_0 = -1.1070023$, $a_1 = 0.005574$, $a_2 = 0.616295$,

$b_0 = -1.1180003$, $b_1 = 0.0056668$, $b_2 = 0.6146161$, h_t = total height (m),

vob = volume over bark (m^3), vub = volume under bark (m^3), $dbhob^2 h_t$ =quadratic diameter over bark multiplied by total height (m^3)

Table 14 *Parameters of total volume over bark and under bark equations*

Equation	Ordinary least squares				Weighted least squares			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
Over Bark								
4.9	0.361151				0.372061			
4.10	0.01462	0.348664			0.0055939	0.3631092		
4.11	-0.0014889*	-2.0044703	0.002789	0.386133	-0.0031354*	-0.5070753*	0.001745	0.350586
4.11#	-0.0198555		0.003176	0.313839	-0.0055556		0.001705	0.332623
4.12	0.00005493	1.58	1.278		0.0000642	1.614	1.196	
4.13	0.008408	0.00003634	1.616	1.365	0.00354**	0.00005097	1.628	1.252
4.14	0.005871*	2.619716			0.017937	2.267522		
Under Bark								
4.9	0.358033				0.368541			
4.10	0.014083	0.346006			0.0053231	0.360023		
4.11	-0.0011616*	-2.058342	0.0027441	0.385793	-0.00291*	-0.557434**	0.00171	0.349983
4.11#	-0.0200218		0.0031415	0.311556	-0.0055705		0.001666	0.330236
4.12	0.00005282	1.58	1.287		0.00006189	1.615	1.204	
4.13	0.008374	0.00003481	1.616	1.375	0.003536	0.000049	1.629	1.26
4.14	0.00459*	2.67637			0.016931	2.316756		

*Note: #=after removing dbhob^2, *= insignificant parameters,**= significant at $\alpha = 0.01$.*

Table 15 *Parameters of transformation equations (total volume over bark and under bark)*

Equation	Over bark				Under bark			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
4.10	-0.871226	0.701515			-0.87829	0.701258		
4.11	-0.760387	0.183839*	0.008198	0.495434	-0.785645	0.176275*	0.008183	0.498728
4.11#	-1.1070023		0.005574	0.616295	-1.1180003		0.0056668	0.614616
4.12	-9.46501	1.70359	1.04815		-9.49729	1.70424	1.05475	
4.13	-3.23936	0.14627	1.01207	0.3042	-3.24038	0.14462	1.01247	0.30635

*Note: #=after removing dbhob^2, *= insignificant parameters*

4.1.4.2. Taper

Taper equations tested in this study were some of the incompatible taper functions and the compatible ones using the selected volume equation (Equation 4.11). The statistical results of the incompatible taper equations and compatible taper functions are shown in Tables 16 and 17. The incompatible taper functions tested in this study had different dependent variables (different *y* values). Therefore, the statistical estimates from Equations 4.17, 4.18 and 4.19 were back transformed to get a *d* value in order to get an equal comparison. Based on Table 16, the standard error of Equation 4.15 was smaller, both in model fitting and validation. The bias of

this equation was the smallest compared to others when we used the validation dataset. The residual plots of Equation 4.15 are shown in Figure 22.

Table 16 *Statistical values of model fitting and validation of incompatible taper functions (over bark)*

Equation	Model fitting*			Validation*		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.15	1.034 (1)	0.14605 (2)	0.8255 (2)	1.0439 (1)	0.0859 (1)	0.8062 (1)
4.16	1.035 (2)	0.1965 (4)	0.8326 (3)	1.0467 (2)	0.18202 (3)	0.84 (3)
4.17	1.0612 (3)	-0.17644 (3)	0.8027 (1)	1.079 (3)	-0.18995 (4)	0.8241 (2)
4.18	1.0753 (4)	0.2964 (5)	0.8663 (4)	1.1152 (4)	0.24252 (5)	0.8939 (4)
4.19	1.1901 (5)	-0.07165 (1)	0.9417 (5)	1.2485 (5)	-0.17073 (2)	0.9146 (5)

*Note: * = the statistical value of d, therefore we get comparison equally among functions.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table 17 *Statistical values of model fitting and validation of compatible taper functions (over bark)*

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.20*	25.09 (8)	-2.37 (1)	17.285 (8)	22.55 (8)	-3.907 (9)	16.192 (8)
4.21	24.21 (4)	-3.363 (4)	16.411 (1)	21.323 (7)	-3.587 (6)	15.298 (4)
4.22*	24.06 (1)	-2.331 (7)	16.5 (5)	21.273 (4)	-2.858 (3)	15.332 (7)
4.23*	24.07 (2)	-2.45 (10)	16.498 (4)	21.19 (3)	-2.933 (4)	15.222 (3)
4.24	24.28 (6)	-3.787 (1)	16.52 (7)	21.315 (5)	-3.652 (7)	15.307 (6)
4.25*	24.3 (7)	-3.792 (4)	16.505 (6)	21.318 (6)	-3.678 (8)	15.302 (5)
4.26	24.07 (2)	-2.758 (7)	16.493 (2)	21.12 (2)	-3.115 (5)	15.131 (2)
4.27	28.43 (10)	-3.745 (1)	20.398 (10)	26.138 (10)	-2.747 (2)	19.097 (10)
4.28	26 (9)	-5.317 (4)	17.922 (9)	23.105 (9)	-4.234 (10)	16.782 (9)
4.29	24.27 (5)	-3.136 (7)	16.496 (3)	20.772 (1)	-2.733 (1)	14.836 (1)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table 18 *Statistical values of do (diameter over bark) from compatible taper functions*

Equation	Model fitting			Validation		
	SEE	MRES	MAB	RMSE	MRES	MAB
4.21	**0.908 (3)**	**-0.089 (5)**	**0.706 (1)**	**0.915 (4)**	**-0.111 (5)**	**0.707 (4)**
4.24	0.908 (2)	-0.089 (6)	0.71 (3)	0.903 (2)	-0.091 (3)	0.705 (3)
4.26	**0.904 (1)**	**-0.066 (3)**	**0.709 (2)**	**0.911 (3)**	**-0.095 (4)**	**0.702 (2)**
4.27	1.154 (6)	0.051 (1)	0.911 (6)	1.18 (6)	0.12 (6)	0.926 (6)
4.28	1.004 (5)	-0.083 (4)	0.784 (5)	1.002 (5)	-0.022 (1)	0.79 (5)
4.29	**0.916 (4)**	**-0.054 (2)**	**0.712 (4)**	**0.903 (1)**	**-0.037 (2)**	**0.699 (1)**

*Note: * = the statistical value of d, hence we get comparison equally among functions.*
Ranking is shown in brackets. (The smallest rank was preferable).

Every polynomial equation had the same dependent variable (d^2), and so they could be compared directly. Of ten polynomial equations, four of them had insignificant parameters,

therefore these equations were discarded. From the six remaining equations, we calculated SEE, MRES and MAE of *do* (diameter over bark) values for these models, so these statistics could also be compared with the incompatible taper equations. The results are shown in Table 18.

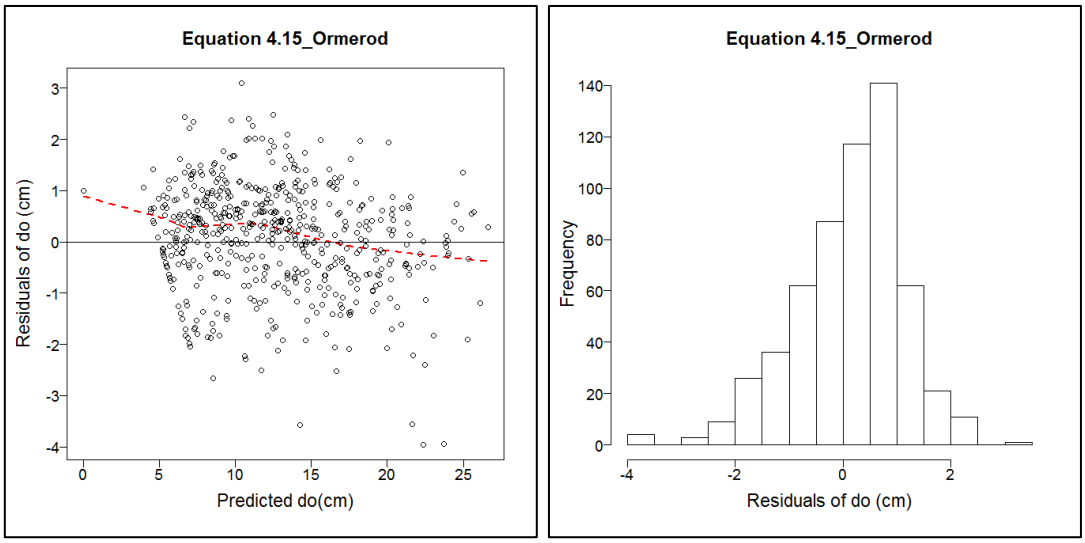

Figure 22 Scatterplot of residuals (left) with lowest smoothing (red dashed line) and histogram of residuals of Equation 4.15

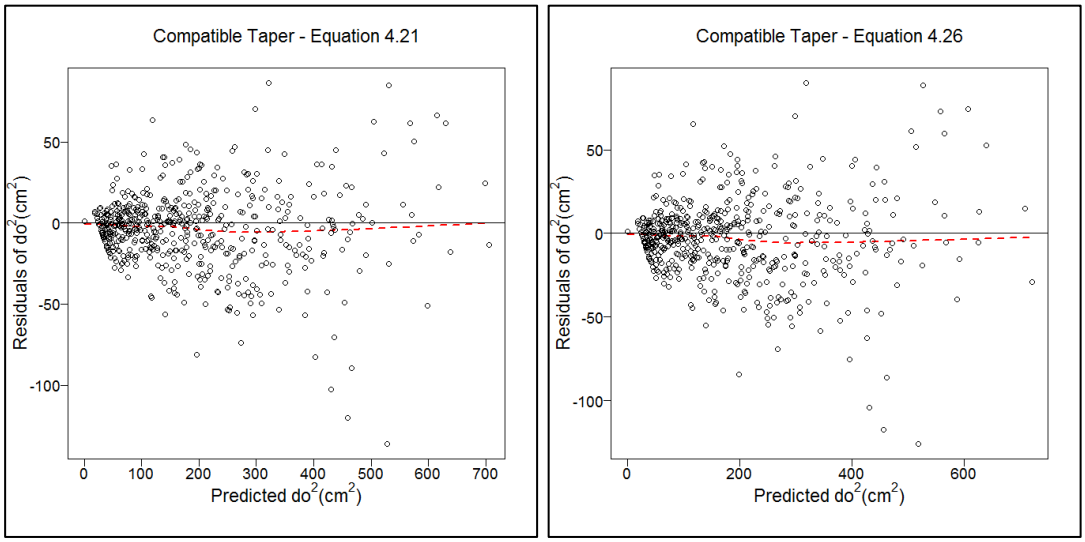

Figure 23 Residual plots of Equation 4.21 (left) and Equation 4.26 in model construction

The three best equations of the polynomial taper functions in model fitting were Equations 4.21, 4.26 and 4.29. If we compared just these three equations, the best equation of compatible taper in model validation was Equation 4.29. However, if we considered the simplicity of an equation, Equation 4.21 was less complex than Equations 4.26 and 4.29, which had five parameters and bigger powers in their parameters. The residual plots of these three equations looked similar. The SEE and MAE of Equation 4.21 only had a small difference compared with Equations 4.29 and 4.26 (see Table 18). The comparison of residual plots between Equation

4.21 and the best equation of model fitting (Equation 4.26) and validation (Equation 4.29) are shown in Figures 23 and 24.

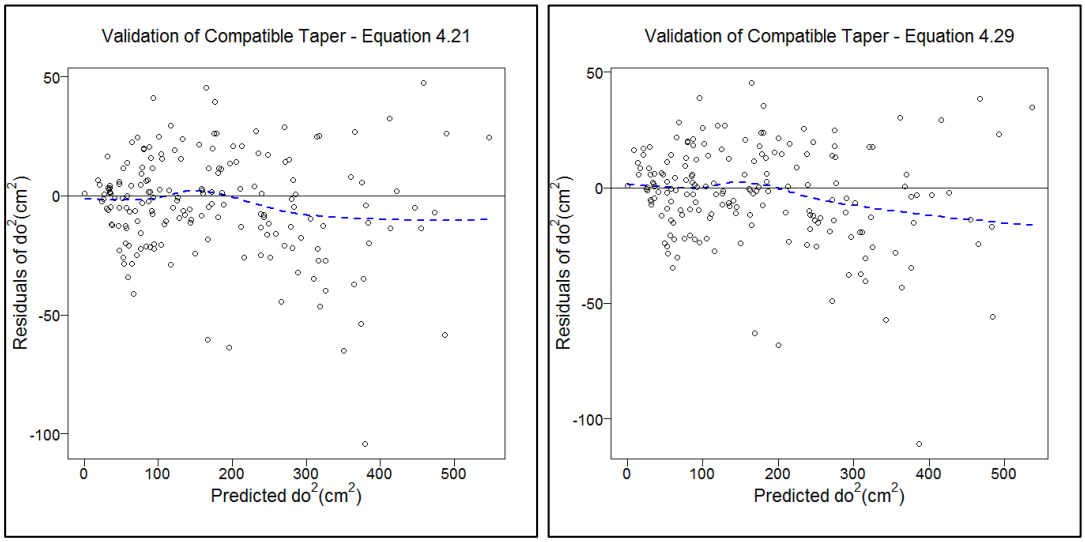

Figure 24 Residual plots of Equation 4.21 (left) and Equation 4.29 (right) in validation

Comparisons of the three best incompatible and compatible taper equations for over bark and under bark are shown in Tables 19 and 20. Overall, the compatible taper functions had better statistical values than the incompatible ones both in model construction and validation. However, we needed to inspect the residual plots to compare the equations from an incompatible taper function (Equation 4.15) and compatible taper function (Equation 4.21). Based on the residual plots in Figure 25, we can see that Equation 4.21 has better residuals (less biased) compared with Equation 4.15. Moreover, the histogram of Equation 4.15 looks more skewed than Equation 4.21 (Figure 26).

Table 19 *Comparison between incompatible and compatible taper functions (over bark).*

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
Incompatible Taper Equations						
4.15	1.034 (1)	0.146(1)	0.8255 (2)	1.0439 (1)	0.0859 (1)	0.8062 (1)
4.16	1.035 (2)	0.1965 (3)	0.8326 (3)	1.0467 (2)	0.182 (2)	0.84 (3)
4.17	1.0612 (3)	-0.1764 (2)	0.8027 (1)	1.079 (3)	-0.19 (3)	0.8241 (2)
Compatible Taper Equations*						
4.21	0.908 (2)	-0.089 (3)	0.7064 (1)	0.915 (3)	-0.1115 (3)	0.707 (3)
4.26	0.904 (1)	-0.066 (2)	0.7089 (2)	0.9115 (2)	-0.0953 (2)	0.7023 (2)
4.29	0.916 (3)	-0.054 (1)	0.7124 (3)	0.9026 (1)	-0.0366 (1)	0.6991 (1)

*Note: * = the statistical value of d, hence we get comparison equally among functions.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table 20 *Comparison between incompatible and compatible taper functions (under bark).*

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
Incompatible Taper Equations						
4.15	1.029 (2)	0.141 (1)	0.820 (2)	1.039 (2)	0.082 (1)	0.8 (1)
4.16	1.027 (1)	0.191 (3)	0.825 (3)	1.037 (1)	0.178 (2)	0.830 (3)
4.17	1.050 (3)	-0.170 (2)	0.793 (1)	1.067 (3)	-0.183 (3)	0.813 (2)
Compatible Taper Equations*						
4.21	0.923 (2)	-0.155 (3)	0.714 (1)	0.936 (3)	-0.183 (3)	0.714 (3)
4.26	0.917 (1)	-0.129 (2)	0.715 (2)	0.929 (2)	-0.162 (2)	0.707 (2)
4.29	0.924 (3)	-0.117 (1)	0.718 (3)	0.908 (1)	-0.1 (1)	0.701 (1)

*Note: * = the statistical value of d, hence we get comparison equally among functions.*
Ranking is shown in brackets. (The smallest rank was preferable).

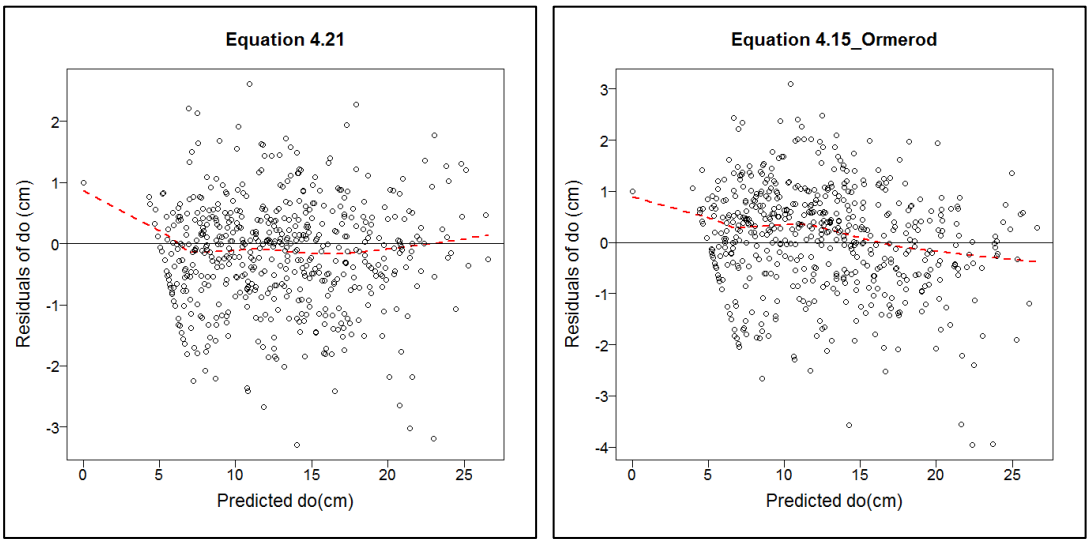

Figure 25 Residual plots of Equation 4.21 (left) and Equation 4.15 (right) in model fitting

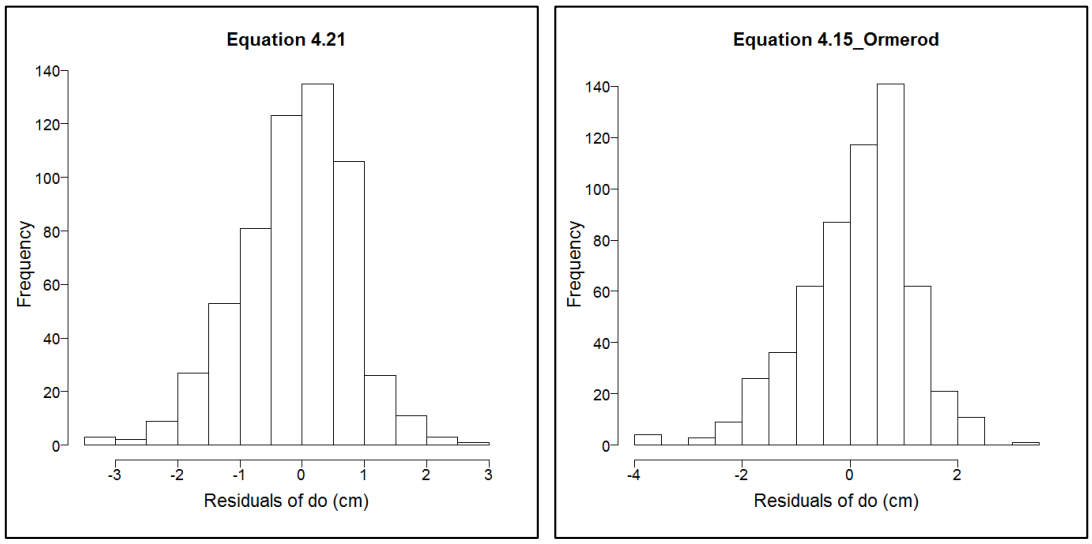

Figure 26 Residuals histogram of Equation 4.21 (left) and Equation 4.15 (right) in model fitting

A previous study of compatible taper equations of *Acacia mangium* was carried out by Krisnawati (2016) in South Sumatra. She found that the best volume equation for this species

was a logarithmic function and a compatible taper equation was derived from it. This study, however, found that the general combined variable with scaled power transformation was the best equation for estimating the volume of *Acacia mangium* in East Kalimantan. The polynomial taper equation gave the best result from this study compared with incompatible taper equations.

The fifth-degree polynomial taper equation gave a better result than the fourth-degree polynomial equation (Equation 4.21). Equation 4.26 was better in model fitting compared with the other two. However, Equation 4.29 gave the best result in validation. If we sum the rank of model fitting and validation, Equation 4.21 gave the worst result of these equations. However, if we look closely, the differences between equations were relatively small. The difference in standard error between Equations 4.21 and 4.26 was only 0.008. The mean absolute error of Equation 4.21 was only 0.008 bigger than Equation 4.29. In addition, Equations 4.26 and 4.29 had more complex powers than Equation 4.21.

Even though most studies (Casnati, 2016; Ounekham, 2009) selected the fifth-degree polynomial taper equation, Kozak et al. (1969) said that complex taper equations gave little advantage for practical purposes. Based on the consideration explained above, the compatible taper equation chosen was Equation 4.21. The taper equation for under bark also gave a similar result, with Equation 4.21 as the one chosen (Appendix B).

$$d^2 = \frac{v_t}{k\, h_t}\left(2\beta_1 z^1 + 3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4\right) \qquad \text{(Equation 4.21)}$$

The merchantable volume (v_{mv}) of any given height can be calculated as follows:

$$v_{mv} = v_t \int_h^0 \left(2\beta_1\left(\frac{h_t - h}{h_t}\right)^1 + 3\beta_2\left(\frac{h_t - h}{h_t}\right)^2 + 4\beta_3\left(\frac{h_t - h}{h_t}\right)^3 + 5\beta_4\left(\frac{h_t - h}{h_t}\right)^4\right) dh$$

and the integration gave this result:

$$v_{mv} = v_t \Big|_h^0 \left[\beta_1\left(\frac{h_t - h}{h_t}\right)^2 + \beta_2\left(\frac{h_t - h}{h_t}\right)^3 + \beta_3\left(\frac{h_t - h}{h_t}\right)^4 + \beta_4\left(\frac{h_t - h}{h_t}\right)^5\right]$$

where:

$z = (h_t\text{-}h)/\, h_t$, $k = (\pi/4\ 10^{-4})$

$\beta_1 = 0.2195$, $\beta_2 = 2.5380$, $\beta_3 = -3.1441$, $\beta_4 = 1.3866$ for over bark

$\beta_1 = 0.3063$, $\beta_2 = 2.2671$, $\beta_3 = -2.8391$, $\beta_4 = 1.2657$ for under bark

d = diameter over bark or under bark at certain h (cm), h_t = total tree height (m), h = distance up the stem from the ground (m), v_{mv} = merchantable volume, v_t = selected volume equation (Equation 4.11)

We ran some simulations using various values of dbhob and total height to get the diameters over bark (do) and under bark (du) by employing Equations 4.11 and 4.21. We found that sometimes du had greater values than do at some upper sections of a stem. If a tree was small, sometimes this trend only occurred in the last section of a stem.

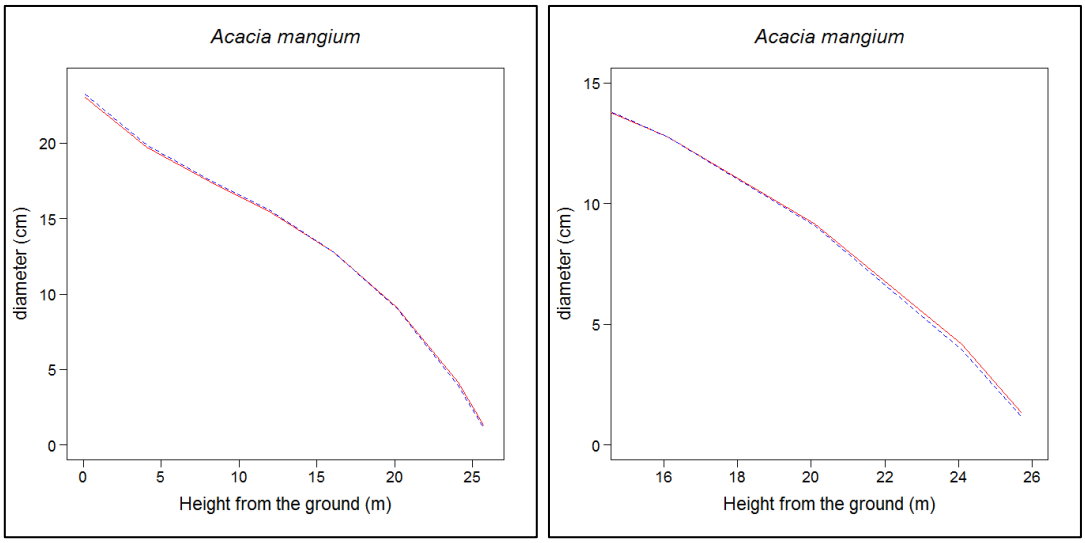

Figure 27 Diameter under bark and over bark simulation from a whole stem (left) and some upper sections of a stem (right) using dbhob =22 cm and total height=26 m.

We inspected the original dataset and found that some of the trees had inconsistencies in bark thickness values. For example, bark thickness at the small-end cross-section of the first log section had different values from the bark thickness at the large-end cross-section of the second section of a log. This, however, is possible, since the bark thickness varies around the tree circumference at a given height, and so the measurements at different points on a tree circumference may result in different values. However, FSS was applying the du calculation described in section IV.3.1. If inconsistency occurs in the bark thickness measurement, a single tree may have two different dus at the same height above ground based on their du calculation.

The values where du exceeded do were very small (less than 0.5 cm from all simulations that we ran), such as the example shown in Figure 27. The total volume and total merchantable volume was not affected by this trend. However, volume over bark of a single section of a log might have a smaller value than the under-bark volume.

Information of under-bark volume was more valuable than over-bark, since the bark of a tree is not used for production. The merchantable volume of a species depends on a specific upper diameter limit, and usually, the tip of trees does not meet this requirement. Based on this consideration, the unexpected trend of the du at upper sections of a log can be ignored.

However, we were much more confident to use the over-bark model, since we did not find any unexpected trend in its original dataset. We recommend that FSS generates a bark-thickness model, and the combination of an over-bark model and bark-thickness model may solve this problem, producing a satisfactory result for merchantable volume under bark.

Using the average total height and dbhob of each age from the volume dataset, the tree profile of *Acacia mangium* is shown in Figure 28. The negative value of diameter in that figure was purposely used to describe a two-dimensional tree and it was just a mirror reflection of the positive value of the diameter at a certain value of *h*.

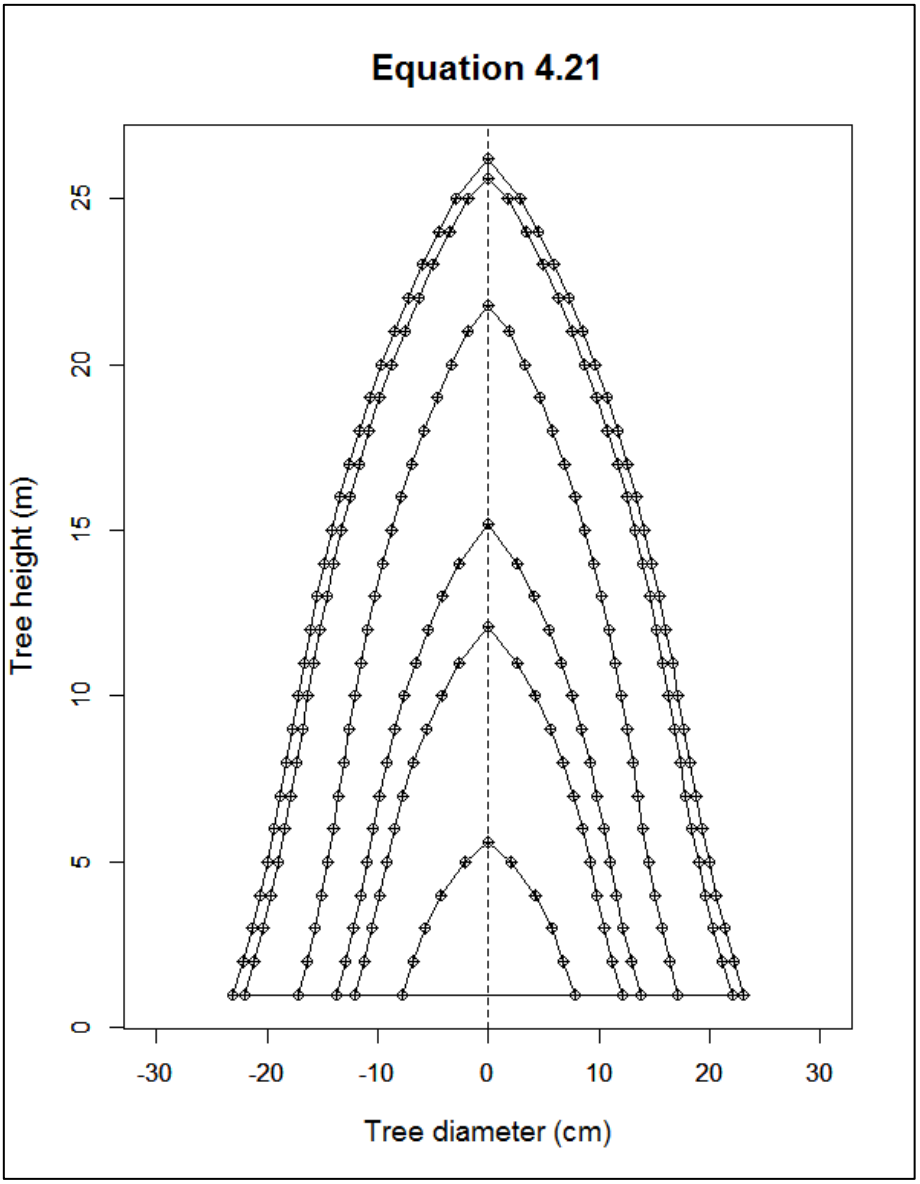

Figure 28 Tree profile of *Acacia mangium* (over bark) at one-metre intervals of log section, using dbhob and total height from inner line to outer line respectively: dbhob = 7.9, 11.8, 13.3, 16.4, 21.7 and 22.9 cm, and total height = 5.6, 12.1, 15.2, 21.8, 25.6 and 26.2 m.

Table 21 *Coefficients of taper equations (over bark)*

Equation	Coefficients							
	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
4.15		0.973842	0.611719					
4.16		1.05078	0.43594	1.57689				
4.17	1.04730	-0.81182	0.49323	-0.65114				
4.18		-1.34667	0.33104					
4.19		2.045						
4.20		0.84937	0.05828*	0.09235*				
4.21		**0.2195**	**2.538**	**-3.1441**	**1.3866**			
4.22		0.694	-0.3636*	3.3228*	-4.8198	2.1666		
4.23		0.3971*	2.0172*	-4.2854*	7.0334*	-6.7962*	2.6327*	
4.24			3.33677	-4.12305	1.78628			
4.25			3.3875	-4.3109	2.0169	-0.09353*		
4.26			4.9716	-13.1765	20.2465	-16.4199	5.3782	
4.27				7.7411	-12.2983	5.5572		
4.28				12.5512	-29.4778	25.9366	-8.01	
4.29				22.216	-80.302	124.705	-92.166	26.548

*Note: *= insignificant parameters*

Table 22 *Coefficients of taper equations (under bark)*

Equation	Coefficients							
	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
4.15		0.96868	0.61041					
4.16		1.04202	0.43785	1.54336				
4.17	1.03982	-0.79974	0.48632	-0.65058				
4.18		-1.32797	0.32303					
4.19		2.047866						
4.20		0.88118	0.003631*	0.11518				
4.21		**0.3063**	**2.2671**	**-2.8391**	**1.2657**			
4.22		0.9061	-1.4008*	5.3358	-6.5798	2.7388		
4.23		0.7459**	-0.1172*	1.2338*	-0.1891*	-2.0936*	1.4151*	
4.24			3.38148	-4.20473	1.82325			
4.25			3.4964	-4.6303	2.3457	-0.2119*		
4.26			5.4333	-15.4704	24.6353	-20.1744	6.5762	
4.27				7.8092	-12.4297	5.6206		
4.28				12.6466	-29.7068	26.1158	-8.0554	
4.29				23.092	-84.638	132.866	-99.013	28.693

*Note: *= insignificant parameters,**=significant at $\alpha = 0.01$*

4.1.5. Conclusions

The volume equation selected for *Acacia mangium* as a result of this study was the scaled power transformation of a general combined variable without using the dbhob^2 parameter

(Equation 4.11). The scaled power transformation performed well in complying with the constant variance of errors assumption in regression and also gave smaller errors compared with the weighted least-squares method. Although the logarithmic function (Equation 4.12) seems slightly better than the general combined variable (Equation 4.11), this equation still presented the inverse of heteroscedasticity.

We recommend using Equation 4.11 to predict volume over bark (vob) and under bark (vub) of *Acacia mangium* within ranges of age from 1 to 6 years, dbhob from 6.1 to 28.1 cm and total height from 4.85 to 29.8 m. Extrapolation outside these ranges is best avoided.

The fourth-degree polynomial taper equation (Equation 4.21) was thought to be applicable to this species for over- and under-bark volume. This equation performed better than the incompatible ones such as Equation 4.15 (Ormerod, 1973) and was simpler than the fifth-degree polynomial equations (Equations 4.26 and 4.29). This fourth-degree equation can be used to predict the diameter at any given height and to predict the merchantable volume. The total volume calculation from this equation will have an identical result to the total volume that was calculated using Equation 4.11.

We found a deficiency in the chosen taper equation at upper sections of a tree. The du that was calculated using this equation sometimes had bigger values than do at upper sections of a tree. However, the exceeded amounts were minimal, and the total volume and total merchantable volume were not affected by these values. Both the total volume and total merchantable volume under bark always give smaller values than over-bark volume.

Estimates of merchantable volume under bark are preferable to over-bark ones. Since the deficiency of the chosen taper equation does not have any impact on the total merchantable volume result, an under-bark taper equation can be applied. However, we recommended that the company generates a bark model for further research. The combination of an over-bark taper model and a bark model might give a satisfactory result for the merchantable volume under bark.

4.2. Height–dbhob Equations

4.2.1. Introduction

Height of a tree plays an important role in forest measurement, such as for estimating tree volume, dominant height and for simulation of tree growth (Mehtätalo, de-Miguel, & Gregoire, 2015). However, height measurements are time-consuming (Gadow & Hui, 1999), costly and often inaccurate (Gadow & Hui, 1999; Husch et al., 2003). Forest managers often opt to create a height–dbhob relationship to minimise the problems that arise from height measurement. We can estimate mean top height of a stand, individual tree volume, tree volume for particular size class distribution and other derivative usage by computing dbhob into a height–dbhob equation.

Height–dbhob equations are ideally derived from pairs of dbhob–height measurements for each inventory in each stand (Gadow & Hui, 1999). However, this specific height–dbhob will produce too many equations for a single species, and it is not practical to be applied in the field. A generalised height–dbhob equation was proposed to overcome this problem by employing stand variables such as site index, basal area/ha, stocking/ha, elevation and other variables. Hence, it will only produce one generalised height–dbhob equation for a single species that can be used across all stands.

This study aimed to create base height–dbhob equations for calculating MTH for a growth dataset and to develop generalised height–dbhob equations that will be useful for tree projections for all species in this study. The base height–dbhob equations are presented in this chapter, and the generalised height–dbhob equations are explained in Chapter VI.

4.2.2. Method

We used repeated individual trees measurements in clean PSP datasets (after removing the unusual increments plots for all species and post-fire data for *Acacia mangium*). Unlike the growth datasets, plots that only had one measurement were also included in this analysis. There were 458 plots with 50,407 repeated measurements data for *Acacia mangium*, 191 plots (18,234 repeated measurements) and 276 plots (25,761 repeated measurements) for *Acacia crassicarpa* and *Eucalyptus pellita* respectively. There are many equations that are used for creating height–dbhob models, but this study only used 18 equations that are shown in Table 23.

Table 23 Equation tested for height–dbhob equations

Model	Author/Name	Equation	Note (references)
Equation 4. 30	Näslund	$h_t = bh + \frac{dbhob^2}{(a + b\ dbhob)^2}$	two-parameter (1,2)
Equation 4. 31	Näslund-Schmidt	$h_t = bh + \left(b + \frac{a}{dbhob}\right)^{-2.5}$	two-parameter (3,4)
Equation 4. 32	Modified Näslund	$h_t = bh + \left(b + \frac{a}{dbhob}\right)^{-5}$	two-parameter (4)
Equation 4. 33	Curtis	$h_t = bh + \frac{a\ dbhob}{(1 + dbhob)^b}$	two-parameter (2,4,5)
Equation 4. 34	Schumacher	$h_t = bh + a\exp(-b\ dbhob^{-1})$	two-parameter (2,4,6)
Equation 4. 35	Meyer	$h_t = bh + a(1 - \exp(-b\ dbhob))$	two-parameter (2,4,7)
Equation 4. 36	Power	$h_t = bh + a\ dbhob^b$	two-parameter (2,8)
Equation 4. 37	Michaelis-Menten	$h_t = bh + \frac{a\ dbhob}{(b + dbhob)}$	two-parameter (2,9)
Equation 4. 38	Wykoff	$h_t = bh + \exp(a - b(dbhob + 1)^{-1})$	two-parameter (2,10)
Equation 4. 39	Garcia	$h_t = a + \frac{b}{(dbhob + 10)}$	two-parameter (4,11)
Equation 4. 40	Prodan	$h_t = bh + \frac{Dbh^2}{a\ dbhob^2 + b\ dbhob + c}$	three-parameter (2,12)
Equation 4. 41	Logistic	$h_t = bh + \frac{a}{1 + b\exp(-c\ dbhob)}$	three-parameter (2,13)
Equation 4. 42	Weibull	$h_t = bh + a(1 - \exp(-b\ dbhob^c))$	three-parameter (2,14,20)
Equation 4. 43	Gompertz	$h_t = bh + a\exp(-b\exp(-c\ dbhob))$	three-parameter (2,15,20)
Equation 4. 44	Sibessen	$h_t = bh + a\ dbhob^{b\ dbhob^c}$	three-parameter (2,16)
Equation 4. 45	Korf	$h_t = bh + a\exp(-b\ dbhob^c)$	three-parameter (2,17,20)
Equation 4. 46	Ratkowsky	$h_t = bh + a\exp\left(\frac{-b}{dbhob + c}\right)$	three-parameter (2,18,20)
Equation 4. 47	Hossfeld IV	$h_t = bh + \frac{a}{1 + \frac{1}{b\ dbhob^c}}$	three-parameter (2,19)

Note: h_t =total height (m), bh=breast height (m), dbhob=Diameter over bark at breast height, a,b,c=parameters References: 1=Näslund (1937) , 2=Mehtätalo et al. (2015) ,3=Schmidt (1967), 4 =Zhao, Mason, and Brown (2006),5=Curtis (1967) , 6= Schumacher (1939), 7=Meyer (1940) 8= Stoffels and van Soest (1953), 9=Michaelis and Menten (1913), 10=Wykoff, Crookston, and Stage (1982), 11= Garcia (1974), 12= Strand (1959), 13=Pearl and Reed (1920), 14=Weibull (1951), 15=Gompertz (1825), 16=Sibbesen (1981), 17=Lundqvist (1957), 18=Ratkowsky (1990), 19 =Peschel (1938), 20 = Krisnawati et al. (2010)

We used a proportion of 2:1 for the fitting and validation dataset. The statistics that we used for comparing models were SEE, MRES, MAE, AIC for model fitting and RMSE, MRES, and MAE for validation. The residual plots also became a major consideration. The smallest

statistic was preferable for the chosen model. After we got a base height–dbhob equation, we used it to create a growth dataset for MTH, by using the actual pairs of diameters and heights for each inventory in each plot. The parameters in this equation varied within a plot. We then added stand variables such as site index (SI), basal area/ha (G), stand stocking (N), age (A), the interaction between stand stocking and age (NA) and elevation (E) into the base height–diameter models to generate a generalised height–diameter equation. We used R software (R Core Team, 2013) for all data analyses.

There are two approaches for modelling generalised height–dbhob functions, the parameter prediction method and the direct method. We used a parameter prediction method (Clutter, 1983; Temesgen & Gadow, 2004; Zhao et al., 2006) for the two-parameter models and a direct approach (Krisnawati et al., 2010; Sharma & Zhang, 2004) for the three-parameter models.

The first method contains two steps after we got the best model: first, we fitted the parameters (*a* and *b*) as a function of stand variables using linear regression, and the second step was refitting the base model with the significant stand variables that affect the *a* and *b* parameters. We randomly selected only one survey datum for parameter fitting in the first step (Zhao, Mason, & Brown, 2006), and hence we only had 458 data for *Acacia mangium*, and 191 and 276 data for *Acacia crassicarpa* and *Eucalyptus pellita* respectively. The linear regression for the parameter estimates by adding stand variables were written below :

$$a = a_0 + a_1E + a_2G + a_3SI + a_4N + a_5A + a_6NA \qquad \text{(Equation 4. 48)}$$

$$b = b_0 + b_1E + b_2G + b_3SI + b_4N + b_5A + b_6NA \qquad \text{(Equation 4. 49)}$$

The second method directly added the stand variables into parameters in the equation by using nonlinear regression. Sharma and Zhang (2004) added stand variables in asymptote, rate and shape parameters, whereas Krisnawati et al. (2010) only added stand variables into one parameter. In this study, we only tested combinations of stand variables (*X*) added into parameter *c*. The example of this method (Equation 6.14) is written:

$$h_t = bh + a\exp(-b\exp(-(c_0 + c_1X_1 + c_2X_2 + c_3X_3 + \cdots + c_nX_n)))$$

where:

a, *b*, *c* = parameters, *E* =Elevation (m), *G* =basal area/ha, *SI* = site index at age 5 years, *N* = stocking (stems/ha), *A* = age (years), *NA* =stocking × age , h_t = total height (m), *bh* = breast height (m), *X* = stand variables

4.2.3. Results and Discussion

We selected the two best equations from each two-parameter and three-parameter model. Equation 4.30 performed the best by having the smallest AIC, MAE and SEE for the two-parameter models, followed by Equation 4.31 for all species. The best equation of the three-parameter height–diameter equations was Equation 4.43 for all species. Equation 4.43 was only slightly different from Equation 4.42 for *Acacia mangium*; however, this equation had a smaller bias compared with Equation 4.42. The statistical values of other models are shown in Appendices C to E. The residual plots of model fitting for the best two-parameter models (Equation 4.30) and the best three-parameter models (Equation 4.43) are shown in Figure 29.

Table 24 *Statistical values of height–dbhob equations*

Equation	Author/ Name	Model fitting				Validation			Note
		MRES	SEE	MAE	AIC	MRES	RMSE	MAE	
					Acacia mangium				
4.30	Näslund	−0.006 (1)	1.986 (4)	1.502 (3)	139890 (1)	−0.036 (3)	1.999 (4)	1.513 (3)	two-parameter
4.31	Näslund-Schmidt	0.032 (4)	1.981 (3)	1.505 (4)	142216 (4)	0.001 (1)	1.998 (3)	1.517 (4)	
4.42	Weibull	−0.009 (3)	1.96 (1)	1.471 (1)	141504 (3)	−0.039 (4)	1.977 (1)	1.482 (1)	three-parameter
4.43	Gompertz	0.007 (2)	1.96 (1)	1.472 (2)	141503 (2)	−0.023 (2)	1.978 (2)	1.483 (2)	
					Acacia crassicarpa				
4.30	Näslund	0.04 (3)	2.482 (3)	1.876 (3)	56749 (3)	0.089 (3)	2.506 (3)	1.898 (3)	two-parameter
4.31	Näslund-Schmidt	0.065 (4)	2.489 (4)	1.883 (4)	56824 (4)	0.113 (4)	2.511 (4)	1.905 (4)	
4.42	Weibull	−0.008 (2)	2.469 (1)	1.867 (2)	56629 (2)	0.042 (1)	2.492 (2)	1.888 (2)	three-parameter
4.43	Gompertz	−0.004 (1)	2.469 (1)	1.865 (1)	56626 (1)	0.046 (2)	2.49 (1)	1.886 (1)	
					Eucalyptus pellita				
4.30	Näslund	0.028 (3)	2.32 (3)	1.821 (3)	78620 (3)	0.026 (3)	2.348 (3)	1.837 (3)	two-parameter
4.31	Näslund-Schmidt	0.05 (4)	2.325 (4)	1.825 (4)	78693 (4)	0.05 (4)	2.351 (4)	1.84 (4)	
4.41	Logistic	−0.009 (2)	2.303 (3)	1.799 (2)	78353 (2)	−0.009 (2)	2.33 (2)	1.816 (2)	three-parameter
4.43	Gompertz	0.006 (1)	2.302 (2)	1.797 (1)	78342 (1)	0.005 (1)	2.329 (1)	1.813 (1)	

Note: Ranking is shown in brackets. This ranking only compared the best four models.

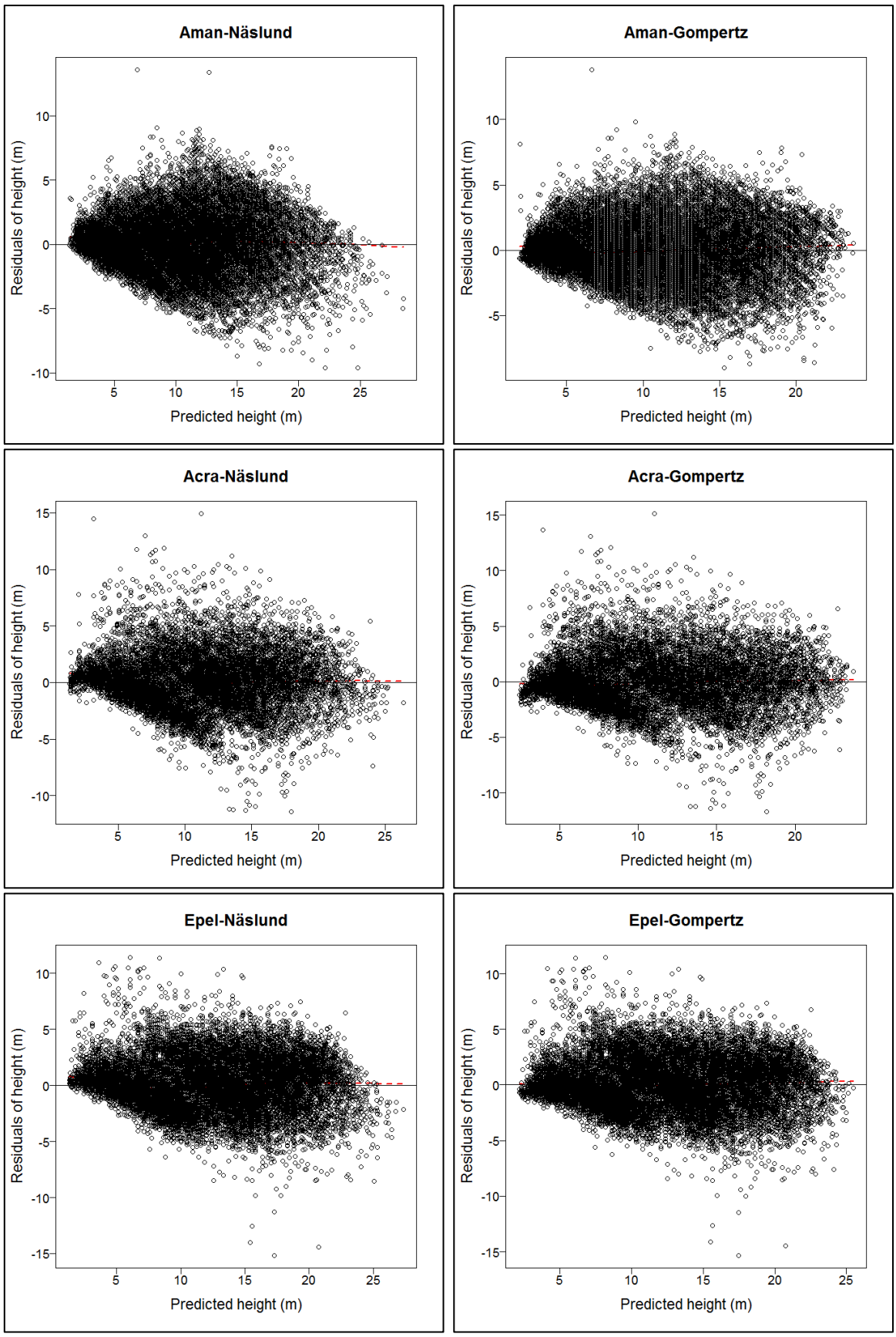

Figure 29 Residual plots of Equations 4.30 (left) and 4.43 (right) in model construction for *Acacia mangium* (top), *Acacia crassicarpa* (middle) and *Eucalyptus pellita* (bottom)

It was clear that three-parameter models more precise by having smaller SEE and MAE values than two-parameter models for all species. However, when the simplicity of a model is considered, the three-parameter model was too complex to be applied in the field. For example, to create a growth dataset from hundreds of plots that came from various stand characteristics, fitting a curve of height–dbhob for each measurement might not be carried out in a single step. On the other hand, two-parameter models are usually easy for fitting of the the height–dbhob curve.

The difference of SEE between two and three parameters was small. It was less than 0.02 for *Acacia crassicarpa* and *Eucalyptus pellita*. For *Acacia mangium*, the difference between Equation 4.30 and 4.43 was only 0.026. Moreover, Equation 4.30 had the smallest AIC and the smallest bias compared with other models for *Acacia mangium*. Zhao et al. (2006) compared 12 two-parameter models and found that the modified Näslund with power value of -5 (Equation 4.32) was chosen for the height–dbhob model of *Pinus radiata* in New Zealand. Krisnawati et al. (2010) compared six equations of three-parameter models for the height–dbhob function for *Acacia mangium* in South Sumatra, Indonesia and they found that the Lundqvist (Equation 4.45) was the best compared with the other five three-parameter models. Mehtätalo et al. (2015) compared 16 functions (7 of two-parameter models and 9 of three-parameter models) for some species in Europe, Asia, North and South America and found that the RMSE of the two-parameter models were higher than or equal to the three-parameter. However, they chose two-parameter equations instead of three-parameter for some species like loblolly pine and *Centrolobium*. In conclusion, we chose Equation 4.30 (two-parameter Näslund with power = -2) for the height–dbhob functions to be applied for *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita*.

CHAPTER V

Stand-Level Growth and Yield Models

5.1. Mean Top Height (MTH)

We tested 23 equations for projecting MTH for *Acacia mangium* and 22 equations *for Acacia crassicarpa* and *Eucalyptus pellita* (see Table 5). The best three equations were then selected for each species. The best three equations of MTH for *Acacia mangium* were Equations 4, 9 and 10. The best three equations for *Acacia crassicarpa* and *Eucalyptus pellita* were Equations 3, 4 and 9 (Table 25).

Table 25 *Statistical values of model fitting for three best equations*

Equation	Author/Name	Statistical values of model fitting			
		MRES	SEE	MAE	AIC
	Acacia mangium				
4	Schumacher/ Polymorphic II	-0.169 (3)	1.561 (2)	1.206 (2)	4505 (2)
9	von Bertalanffy –Richards Polymorphic I	-0.048 (2)	1.315 (1)	1.022 (1)	4090 (1)
10	von Bertalanffy –Richards Polymorphic II	-0.016 (1)	1.584 (3)	1.224 (3)	4540 (3)
	Acacia crassicarpa				
3	Schumacher/ Polymorphic I	-0.01 (2)	1.672 (3)	1.357 (3)	2461 (3)
4	Schumacher /Polymorphic II	-0.007 (1)	1.536 (2)	1.222 (2)	2355 (2)
9	von Bertalanffy –Richards Polymorphic I	-0.022 (3)	1.363 (1)	1.08 (1)	2203 (1)
	Eucalyptus pellita				
3	Schumacher/ Polymorphic I	0.026 (2)	2.178 (3)	1.748 (3)	4033 (3)
4	Schumacher /Polymorphic II	0.056 (3)	2.093 (2)	1.628 (2)	3961 (2)
9	von Bertalanffy –Richards Polymorphic I	0.005 (1)	2.013 (1)	1.571 (1)	3889 (1)

Note: Ranking is shown in bracket. This ranking only compared the best three models.

Equation 9 (von Bertalanffy-Richard Polymorphic I) was the best model for all species. SEE was around 1.3 and MAE was less than 1.1 for *Acacia mangium and Acacia crassicarpa*. SEE and MAE for *Eucalyptus pellita* are 2 and 1.6 respectively. Equation 9 also had the smallest AIC number compared with others model for all species. The residual plots of this equation seemed to have a constant variance for all species, and plots showed minimal bias (Figure 30). Despite some outliers that are shown in Figure 30 for *Eucalyptus pellita*, the histograms of residuals look close to normal. The statistical values of other models are shown in Appendices C-E.

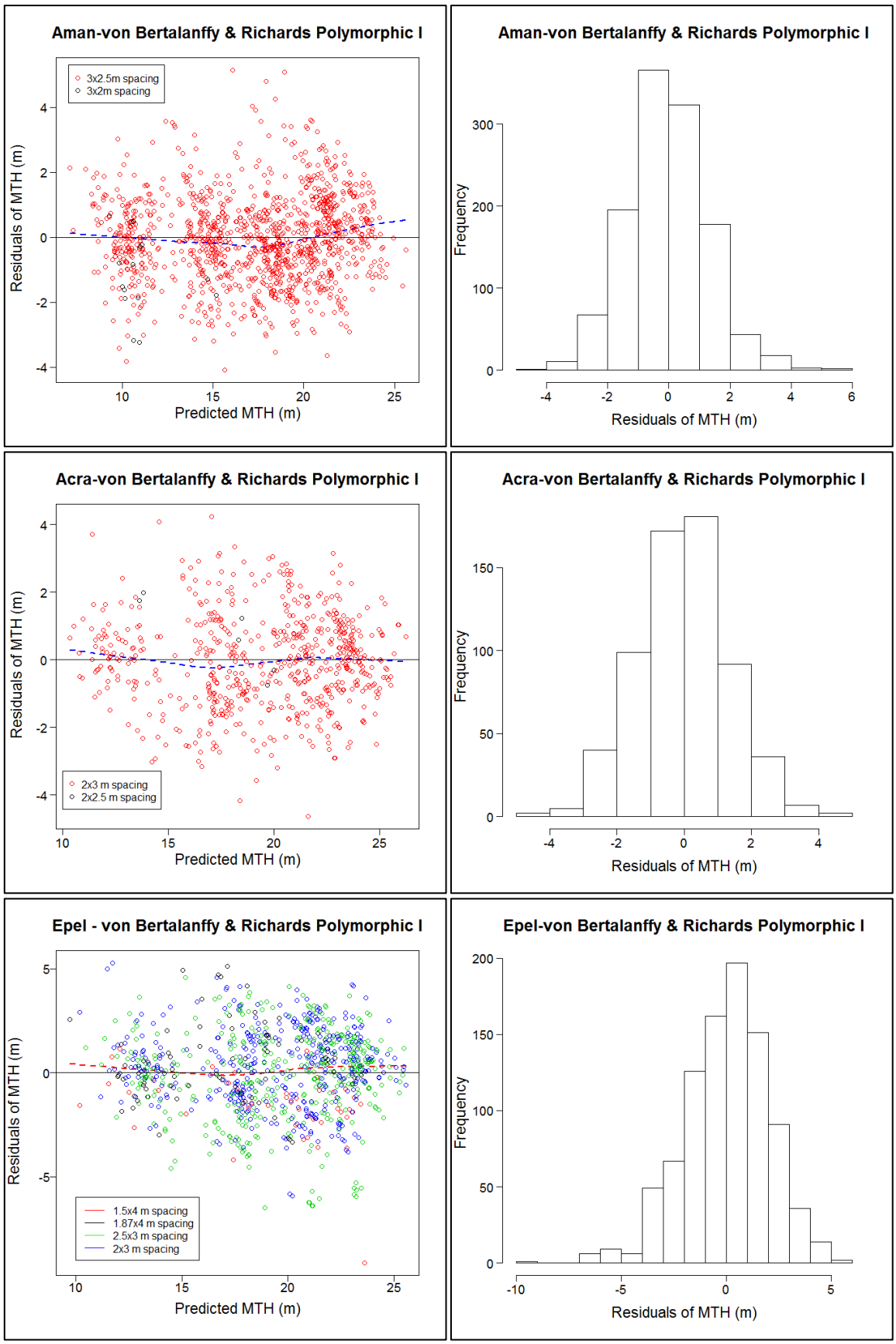

Figure 30 Scatterplots (left) with lowess smoothing dashed line) and histogram (right) of residuals of Equation 9 in model construction for: *Acacia mangium* (top), *Acacia crassicarpa* (middle) and *Eucalyptus pellita* (bottom).

Validation was carried out for the best three equations for each species. The result of the validation process also showed that Equation 9 was superior to the other two equations for all species. Based on the MRES value, Equation 9 both in model fitting and validation produced slight overestimates for *Acacia mangium* and *Acacia crassicarpa,* and underestimates for *Eucalyptus pellita*.

Parameter checking was conducted using one data interval from each plot that was randomly selected (see Table 4). The result for this step found that all parameters in Equation 9 were significant for all species. The residual plots for this equation can be seen in Figure 30-31 and the projections of this equation using site index at age 5 years are shown in Figures 32–33.

Several studies (Casnati, 2016; Methol, 2001) reported that von Bertalanffy Polymorphic I was the best for height modelling. In addition, Torres Vélez and Del Valle (2007) also found that this equation was the best for projecting the height of *Acacia mangium* in Colombia. Gadow and Hui (1999) explained that the von Bertalanffy-Richard Polymorphic I is more appropriate for use in long-term projection by having an asymptote (*b*) in its equation. The *b* coefficient can be directly used as a site index (Jansen, 1996).

Table 26 *Statistical values of validation for three best equations for MTH*

Equation	Author/Name	Statistical value of validation		
		MRES	**SEE**	**MAE**
	Acacia mangium			
4	Schumacher/ Polymorphic II	0.127 (3)	1.759 (2)	1.316 (2)
9	von Bertalanffy –Richards Polymorphic I	-0.006 (1)	1.471 (1)	1.081 (1)
10	von Bertalanffy –Richards Polymorphic II	0.12 (2)	1.791 (3)	1.347 (3)
	Acacia crassicarpa			
3	Schumacher/ Polymorphic I	-0.354 (2)	1.673 (3)	1.329 (3)
4	Schumacher /Polymorphic II	-0.371 (3)	1.639 (2)	1.323 (2)
9	von Bertalanffy –Richards Polymorphic I	-0.327 (1)	1.45 (1)	1.178 (1)
	Eucalyptus pellita			
3	Schumacher/ Polymorphic I	0.191 (2)	2.183 (2)	1.693 (3)
4	Schumacher /Polymorphic II	0.247 (3)	2.218 (3)	1.691 (2)
9	von Bertalanffy –Richards Polymorphic I	0.153 (1)	2.126 (1)	1.612 (1)

Note: Ranking is shown in bracket. This ranking only compared the best three models.

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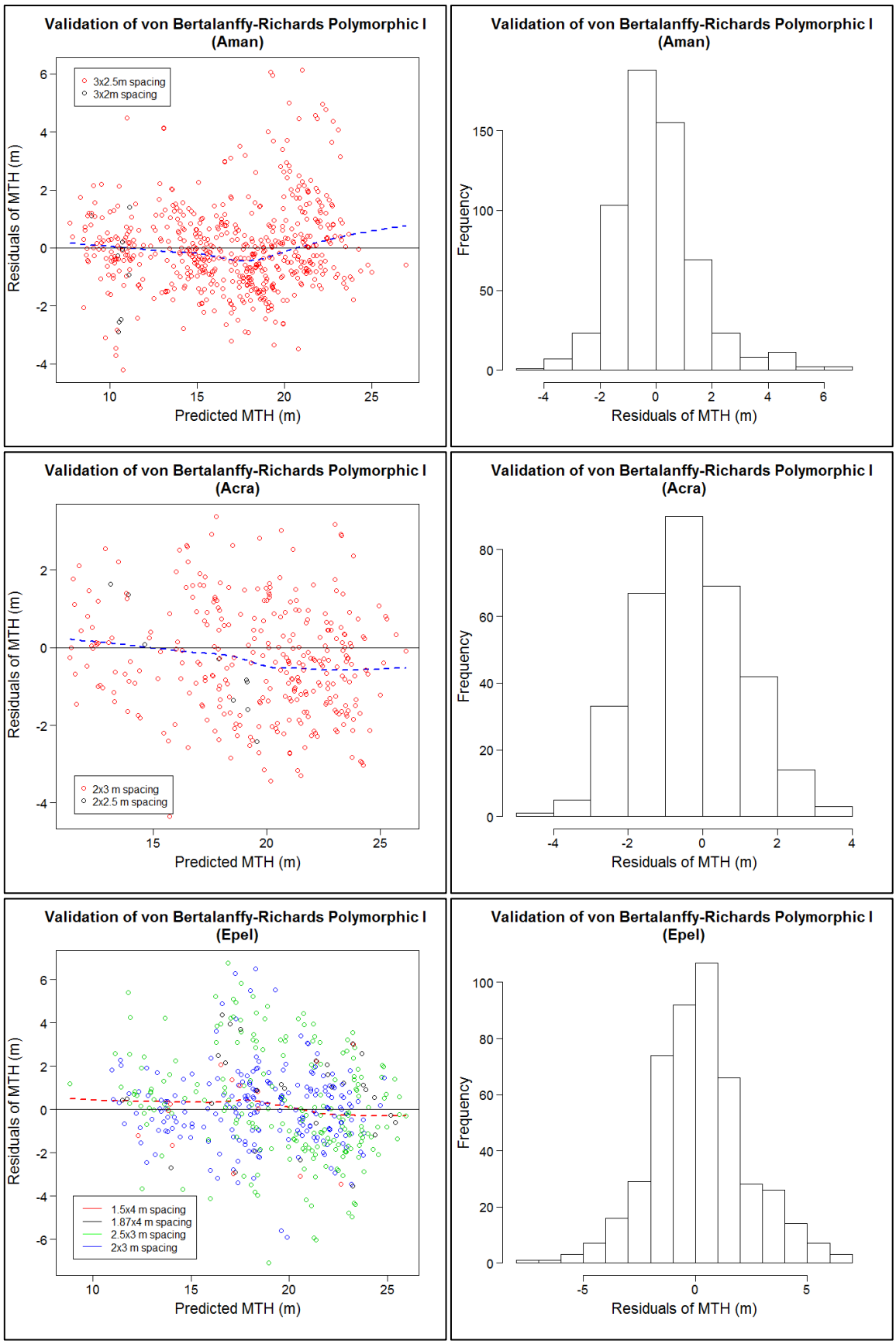

Figure 31 Scatterplots (left) with lowess smoothing (dashed line) and histogram (right) of residuals of Equation 9 in validation for: *Acacia mangium* (top), *Acacia crassicarpa* (middle) and *Eucalyptus pellita* (bottom).

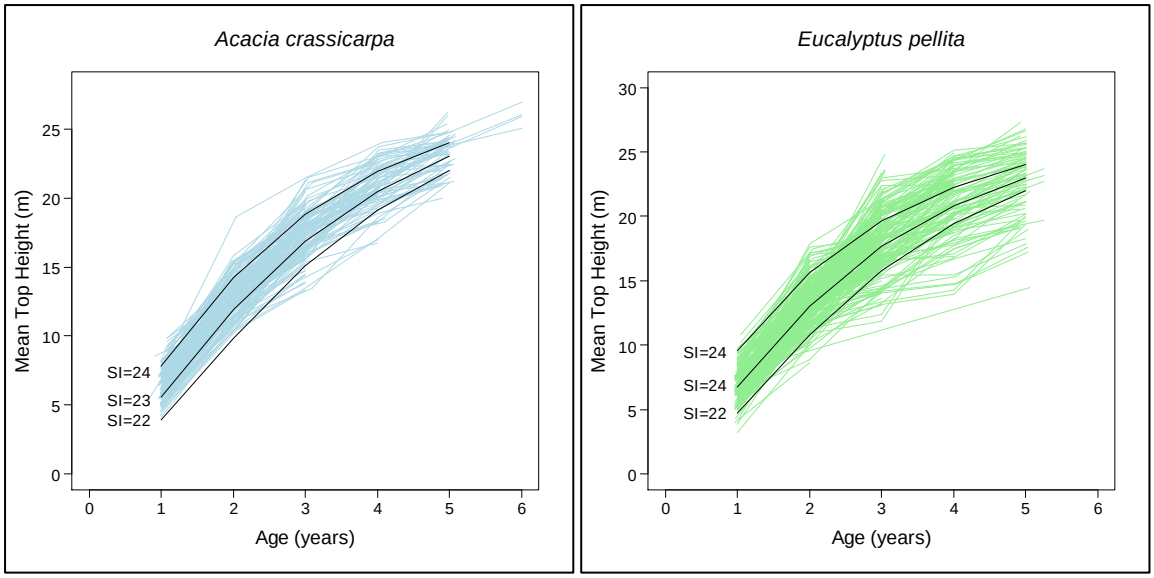

Figure 32 The von Bertalanffy–Richards Polymorphic I (Equation 9) projections when fitted to MTH dataset using Site Index 22, 23 and 24 metres at age five years for *Acacia crassicarpa* (left) and *Eucalyptus pellita* (right).

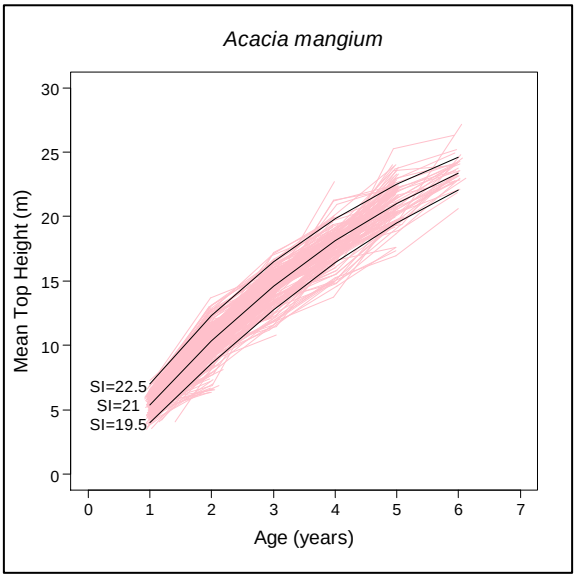

Figure 33 The von Bertalanffy–Richards Polymorphic I (Equation 9) projections (backward and forward) when fitted to the actual dataset of MTH using Site Index 19.5, 21 and 22.5 metres at age five years for *Acacia mangium*.

5.1.1. The Previous Equation of MTH for *Acacia mangium*

The previous equation used in this study area for *Acacia mangium* is

$$MTH = b\,(1 - e^{-c\,T})^{d}$$

where:

$b = 33.92242$, $c = 0.318277$, $d = 1.561575$, and T = age (years).

Since it was a yield equation, it could not be compared with the chosen growth equation of MTH. However, given a yield dataset of MTH, we can see the residual plots and histogram of this equation in Figure 34. The residual graph is clearly biased and the histogram was skewed. Based on Draper and Smith (1966), this residual plot explained the error in the analysis (the departure from the fitted equation is systematic, when positive residuals correspond to low fitted values and negative residuals to high fitted values). It can be caused by wrongly omitting an asymptote in the model (Draper & Smith, 1966). The value of SEE for this equation is 1.9 and MAE for this equation was 1.51. Equation 9 (von Bertalanffy-Richard Polymorphic I) is actually a growth form from the yield equation above. However, instead of keeping the *d* parameter, the asymptote *b* of this equation is maintained for growth equation (Gadow & Hui, 1999)

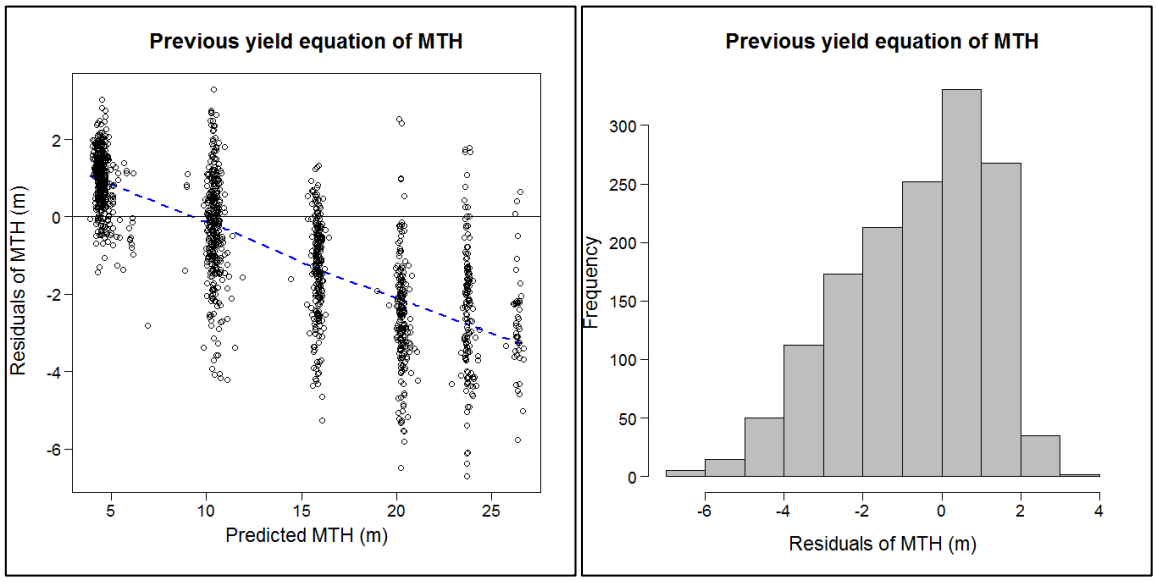

Figure 34 The scatterplot and histogram of residuals from the previous equation of MTH using the actual yield dataset.

5.2. Basal Area (G)

Schumacher Polymorphic II and III (Equations 4 and 5) were the best three equations for basal area models for all species (see Table 27). However, when we tried to make a projection, Schumacher Polymorphic III (Equation 5) could only make a forward projection. For instance, if we have an inventory data at age three years, this equation can only do a projection from the inventory age (three years) to the end of the period (five years). This equation could not project for ages one or two years (a backward projection), as we can see in Figure 35 for *Acacia crassicarpa*.

Table 27 *Statistical values of model fitting for three best equations for basal area*

Equation	Author/Name	Statistical values of model fitting			
		MRES	SEE	MAE	AIC
	Acacia mangium				
4	Schumacher/ Polymorphic II	-0.096 (3)	2.212 (1)	1.675 (1)	5346 (1)
5	Schumacher /Polymorphic III	-0.074 (3)	2.227 (2)	1.689 (2)	5362 (2)
9	von Bertalanffy –Richards Polymorphic I	0.009 (1)	2.335 (3)	1.803 (3)	5477 (3)
	Acacia crassicarpa				
3	Schumacher/ Polymorphic I	-0.06 (2)	2.348 (2)	1.842 (2)	2894 (2)
4	Schumacher /Polymorphic II	-0.062 (3)	2.35 (3)	1.843 (3)	2895 (3)
5	Schumacher /Polymorphic III	-0.001 (1)	2.344 (1)	1.837 (1)	2892 (1)
	Eucalyptus pellita				
4	Schumacher /Polymorphic II	-0.046 (2)	2.512 (1)	1.871 (1)	4296 (1)
5	Schumacher /Polymorphic III	-0.209 (3)	2.695 (3)	2.03 (3)	4425 (3)
21	Hossfeld Polymorphic	-0.011 (1)	2.597 (2)	1.943 (2)	4357 (2)

Note: Ranking is shown in brackets. This ranking only compared the best three models.

Table 28 *Statistical values of validation for three best equations for basal area*

Equation	Author/Name	Statistical values of model validation		
		MRES	SEE	MAE
	Acacia mangium			
4	Schumacher/ Polymorphic II	-0.112 (3)	2.491 (1)	1.911 (1)
5	Schumacher /Polymorphic III	-0.105 (2)	2.527 (3)	1.911 (1)
9	von Bertalanffy –Richards Polymorphic I	-0.007 (1)	2.525 (2)	2.021 (3)
	Acacia crassicarpa			
3	Schumacher/ Polymorphic I	-0.045 (3)	2.468 (1)	1.917 (1)
4	Schumacher /Polymorphic II	-0.041 (2)	2.474 (3)	1.917 (2)
5	Schumacher /Polymorphic III	0.017 (1)	2.468 (2)	1.922 (3)
	Eucalyptus pellita			
4	Schumacher /Polymorphic II	0.217 (2)	2.365 (2)	1.815 (2)
5	Schumacher /Polymorphic III	0.003 (1)	2.342 (1)	1.806 (1)
21	Hossfeld Polymorphic	0.245 (3)	2.513 (3)	1.938 (3)

Note: Ranking is shown in bracket. This ranking only compared the best three models.

It was clear that Schumacher Polymorphic II (Equation 4) was superior in terms of SEE and MAE values compared with other equations for *Acacia mangium* and *Eucalyptus pellita* from the model construction process. For *Acacia crassicarpa*, due to the limited projection capability of Equation 5, the best equation was Equation 3. However, if we look closely, Equation 3 is only slightly better compared with Equation 4. The residual plots between these two equations

also looked similar (Figures 36 and 37). Moreover, bias (MRES) of the validation process of Equation 4 gave a better result than Equation 3.

The mean residuals (MRES) of Equation 4 showed bias from −0.04 to −0.1 (overestimates) for all species. Standard errors (SEE) ranged from 2.2 to 2.5 m^2/ha for all species. Basal area projections for this equation for all species are shown in Figures 38 to 39. Several studies (Berrill, 2004; Methol, 2001; Zhao, 1999) also found that the two-parameter Schumacher Polymorphic equation was the best model for basal area projection for some species. Based on these considerations, Equation 4 was chosen to project basal area for all species.

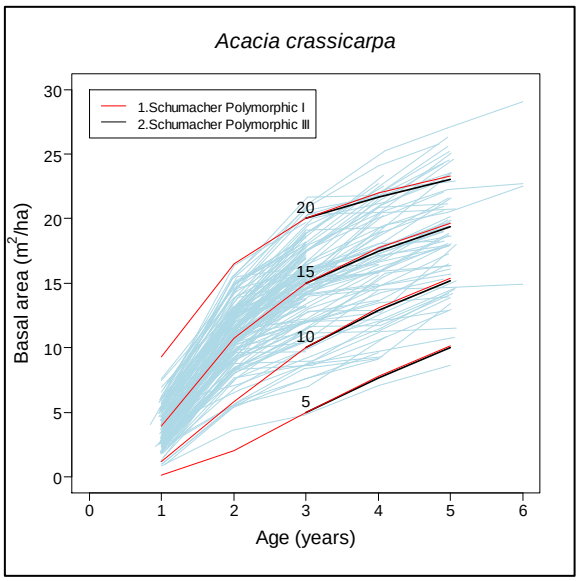

Figure 35 Comparison between Schumacher Polymorphic I (Equation 3) and Schumacher Polymorphic III (Equation 5) when fitted to the actual dataset of basal area using basal area values (5, 10,15 and 20 m^2/ha) at age three years for *Acacia crassicarpa.*

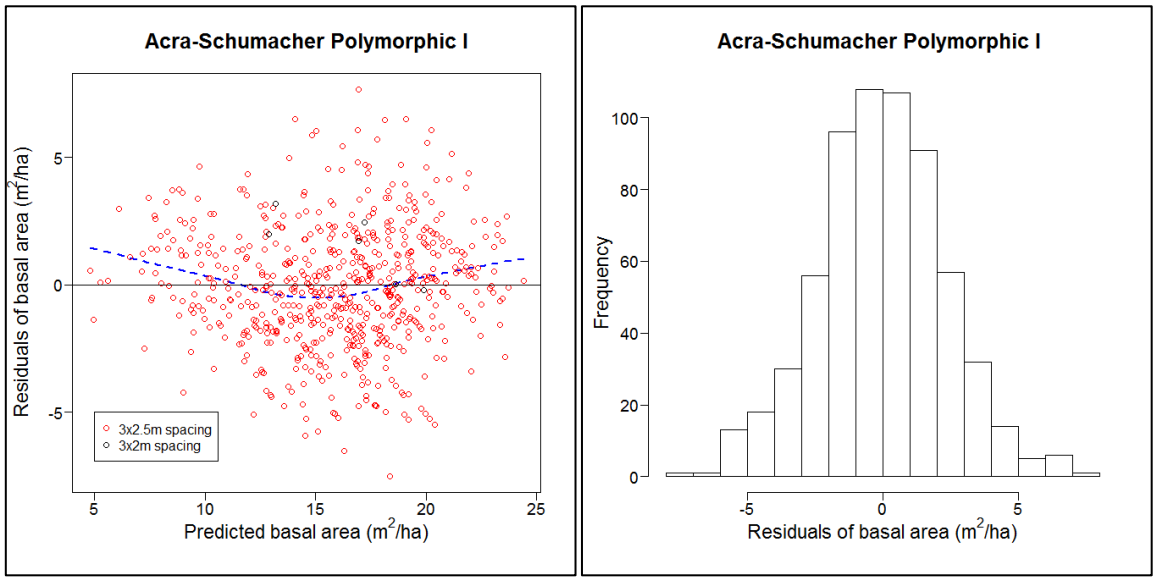

Figure 36 Scatterplots (left) with lowess smoothing (dashed line) and histogram (right) of residuals of Equation 3 in model construction for *Acacia crassicarpa*.

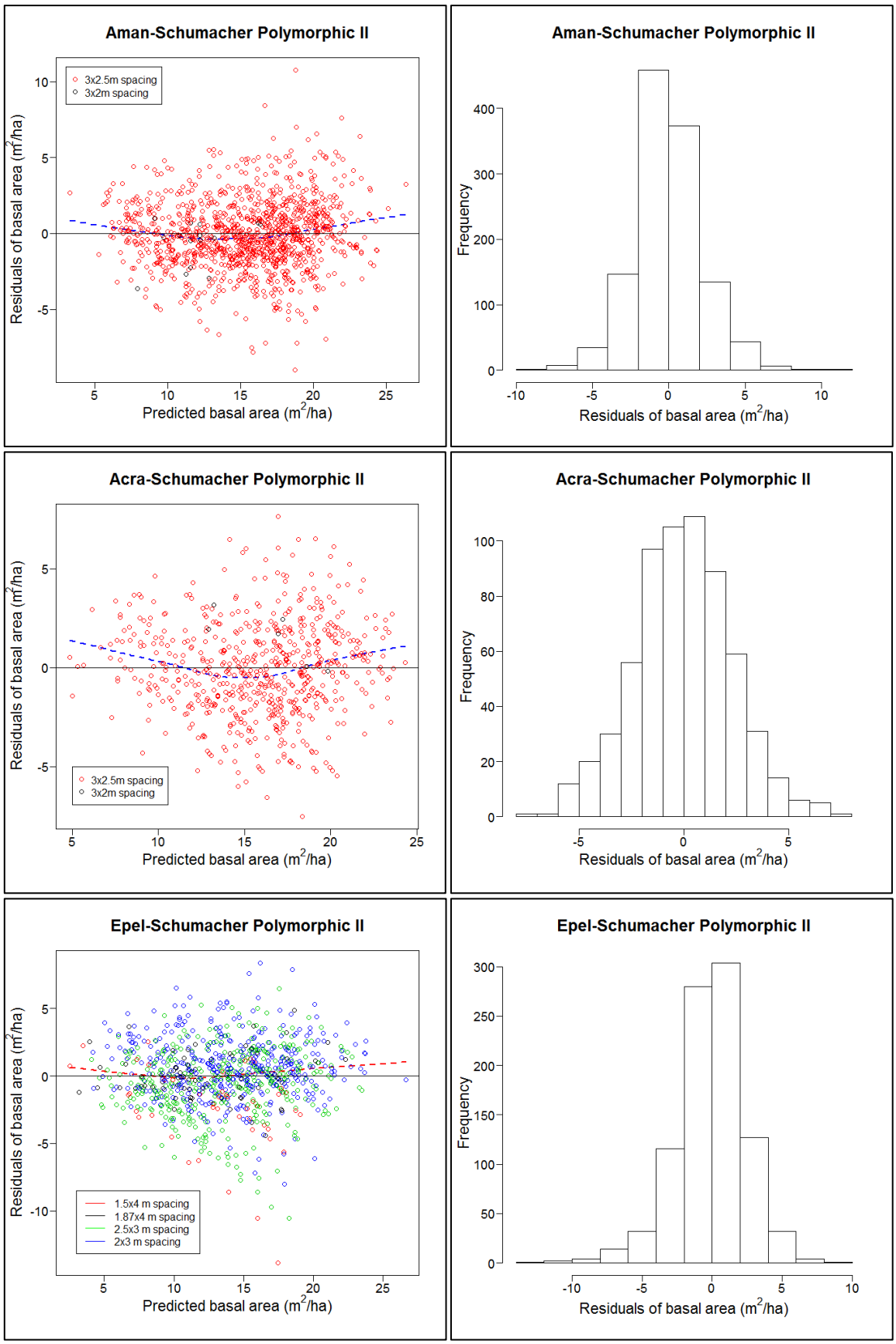

Figure 37 Scatterplots (left) with lowess smoothing (dashed line) and histogram (right) of residuals of Equation 4 in model construction for: *Acacia mangium* (top), *Acacia crassicarpa* (middle) and *Eucalyptus pellita* (bottom).

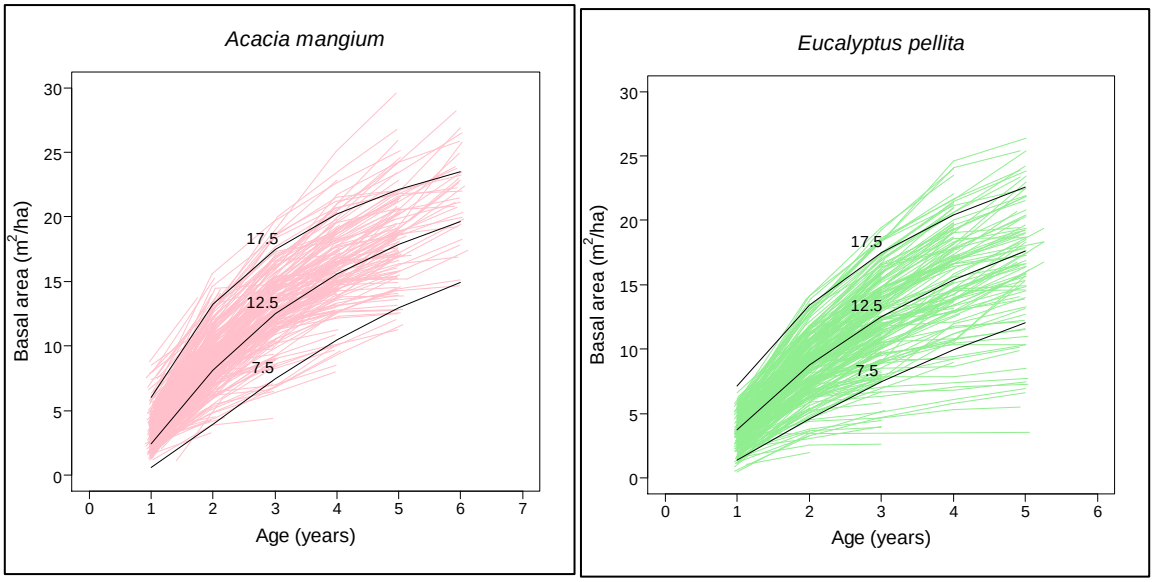

Figure 38 Schumacher Polymorphic II (Equation 4) projections when fitted to the actual dataset of basal area using basal area values (7.5, 12.5 and 17.5 m^2/ha) at age three years for *Acacia mangium* (left) and *Eucalyptus pellita* (right).

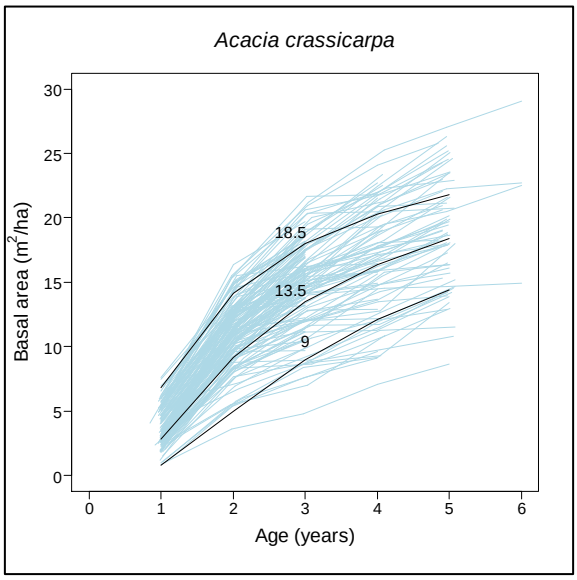

Figure 39 Schumacher Polymorphic II (Equation 4) projections when fitted to the actual dataset of basal area using basal area values (9, 13.5 and 18 m^2/ha) at age three years for *Acacia crassicarpa*.

5.3. Mortality (N)

5.3.1. Comparison of Mortality Approaches

Three approaches of growth interval were compared to get a better mortality estimation. The first approach was the all-possible growth interval which allows the combination of various growth intervals (one year, two years, three years and so on). For a single PSP that has been measured six times, the possible combinations of the interval can be up to 15 combinations. The second approach was two-step regressions which used a constant period of interval, and

we used a one-year interval for this study. Hence, with a PSP measured six times, data for only five growth intervals are produced for this method. The last approach is the longest growth interval, which permitted a single PSP to have only one interval data. For example, a six-times-measured PSP will have the same number of data (only one data) with two times-measured PSP, the longest one was chosen for this method.

For method comparison, we chose the three best equations for each method based on the smallest AIC. However, the models that have insignificant parameters were not chosen even though they had a small value of AIC. Parameter checking was conducted for all-possible interval data and two-step regression to avoid pseudoreplication. The longest interval approach did not need parameter checking since no replication occurred in this dataset. We used the all-possible interval validation datasets for method comparison.

After removing the insignificant models, we chose equations 7, 12 and 17 for the all-possible interval approach for *Acacia mangium*. However, these equations had insignificant parameters; hence we chose equations 2, 22 and 30. Rejection in the parameter checking process for equation 29, led us to choose equations 11, 22 and 30 for *Acacia crassicarpa*. The chosen equations for all-possible intervals for *Eucalyptus pellita* were equations 4, 17 and 27 after rejecting equation 22 in the parameter checking process.

The parameter checking of one-year interval data showed a significant result for the three best equations for *Acacia mangium* and *Acacia crassicarpa.* Therefore, we chose equations 2, 8, 30 and equations 2, 11, 22 for *Acacia mangium* and *Acacia crassicarpa* respectively. Since there were some rejections in parameter checking for equations 2, 8 and 22, we chose equations 4, 13 and 15 for *Eucalyptus pellita*. The thing to remember in two-step regression is that we only used the data with mortality for generating the model. The second step for this approach is creating a probability function of mortality (p). We used a generalised linear model (GLM) with binomial family function for predicting the *logit p* (Equation 5.1).

$$logit\,(p) = log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_n X_n \qquad \text{(Equation 5. 1)}$$

Equation 2.2 then can be written as

$$p = \frac{1}{(1+\exp(-logit\,(p)))}\ , \qquad \text{(Equation 5. 2)}$$

The response (Y) variable of Equation 5.1 was denoted as the event values (0,1) when 1 means mortality occurred and 0 means no mortality occurred during a constant period (Woollons,

1998). However, the explanatory (*X*) variables were specific for each species. We found that starting age (T1), stocking at starting age (N1) and site index at age 5 years (SI) gave significant results for Equation 5.1 for *Acacia mangium*. For *Eucalyptus pellita,* only T1 and N1 variables gave significant results and only one variable, T1, gave significant result for Equation 5.1 for *Acacia crassicarpa.* The stocking at the ending age of the projection (N2) will be adjusted using Equation 2.3.

Table 29 *Parameters of logit p equation (Equation 5.1) for each species*

Coefficients	***Acacia mangium***	***Acacia crassicarpa***	***Eucalyptus pellita***
Intercept	- 13.69	3.5581	2.6378932
N1	0.001586		-0.0006226
T1	0.6976	-0.4317	-0.4205577
SI	0.6243		

General information about the three best equations for each method for each species is shown in Tables 30-32. From these tables, we can see that the two-step regression approach had the smallest standard error (SEE) compared with the two other approaches. However, we could not compare these using these statistics since the datasets for each method were different from each other. Using a validation dataset from all-possible intervals, we tried to compare those approaches. (Tables 33–35). More details about the statistical results of these approach for overall models on each species are written in Appendices C, D and E.

Table 30 *Statistical values of model fitting for three best equations of different mortality approach for* Acacia mangium

Equation	**Author/Name**	**Statistical values of model fitting**			
		MRES	**SEE**	**MAE**	**AIC**
	All-possible intervals				
2	Lundqvist–Korf (Schumacher Anamorphic II)	-4.119 (2)	132.5 (1)	95.9 (1)	15226 (1)
22	Hossfeld IV	-4.543 (3)	133 (2)	96.2 (2)	15235 (2)
30	Exponential decay anamorphic	-2.401 (1)	133.6 (3)	98.6 (3)	15245 (3)
	*One-year interval (two-step regression)**				
2	Lundqvist–Korf (Schumacher Anamorphic II)	-3.193 (3)	112.6 (2)	79.9 (3)	6208 (2)
8	von Bertalanffy–Richards Anamorphic	-3.163 (2)	112.4 (1)	79.7 (2)	6207 (1)
30	Exponential decay anamorphic	-2.513 (1)	112.8 (3)	79.2 (1)	6209 (3)
	Longest interval				
16	Gompertz Polymorphic I	-4.697 (1)	173.4 (1)	138.7 (1)	2818 (1)
27	Anamorphic III	-4.934 (2)	176.1 (2)	140.9 (2)	2823 (2)
28	Anamorphic IV	-5.46 (3)	176.6 (3)	141.4 (3)	2825 (3)

Note: Ranking is shown in bracket. This ranking only compared the best three models.
**=The value before adjustment (using only a dataset when mortality occurred).*

Table 31 *Statistical values of model fitting for three best equations of different mortality approaches for* Acacia crassicarpa

Equation	Author/Name	Statistical values of model fitting			
		MRES	SEE	MAE	AIC
	All-possible intervals				
11	Weibull	3.381 (3)	176 (1)	135.8 (1)	8386 (1)
22	Hossfeld IV	3.312 (2)	177.3 (2)	136.8 (2)	8395 (2)
30	Exponential decay anamorphic	2.659 (1)	178.1 (3)	137.5 (3)	8400 (3)
	*One-year interval (two-step regression)**				
2	Lundqvist–Korf (Schumacher Anamorphic II)	1.71 (1)	138.6 (3)	107.3 (3)	3459 (3)
11	Weibull	2.334 (2)	134.9 (1)	104.8 (1)	3444 (1)
22	Hossfeld IV	3.047 (3)	137 (2)	106.2 (2)	3452 (2)
	Longest interval				
11	Weibull	3.575 (2)	235.8 (2)	189.7 (1)	1312 (2)
20	Hossfeld Anamorphic	-17.18 (3)	238.3 (3)	195.6 (3)	1314 (3)
30	Exponential decay anamorphic	1.644 (1)	235.5 (1)	191.5 (2)	1310 (1)

Note: Ranking is shown in bracket. This ranking only compared the best three models.
**=The value before adjustment (using only a dataset when mortality occurred).*

Table 32 *Statistical values of model fitting for three best equations of different mortality approaches for* Eucalyptus pellita

Equation	Author/Name	Statistical values of model fitting			
		MRES	SEE	MAE	AIC
	All-possible intervals				
4	Schumacher/ Polymorphic II	0.463 (1)	199.3 (1)	147.7 (1)	12317 (1)
17	Logistic I	-4.328 (2)	201.7 (3)	151.1 (3)	12339 (3)
27	Anamorphic III	-4.912 (3)	201.4 (2)	150.7 (2)	12335 (2)
	*One-year interval (two-step regression)**				
4	Schumacher/ Polymorphic II	-7.121 (2)	135.4 (1)	100.6 (1)	3990 (1)
15	Gompertz	-6.627 (1)	136.9 (3)	104.9 (3)	3997 (3)
22	Hossfeld Polymorphic	-8.672 (3)	136.1 (2)	101.5 (2)	3993 (2)
	Longest interval				
27	Anamorphic III	0.181 (1)	269.1 (1)	217.4 (2)	2290 (1)
28	Anamorphic IV	-1.173 (2)	269.8 (3)	218.2 (3)	2290 (3)
30	Exponential decay anamorphic	-10.465 (3)	269.4 (2)	215.3 (1)	2290 (2)

Note: Ranking is shown in bracket. This ranking only compared the best three models.
**=The value before adjustment (using only a dataset when mortality occurred).*

Two-step regression models for *Acacia mangium* also exhibited the smallest values of bias (MRES) compared with other methods (Table 33). Equation 2 of the two-step regression had a reduction of SEE for about 2 stems/ha compared to the all-possible interval approach. Meanwhile, Equation 30 increased by about 1 stem/ha of SEE. The value for MAE of the two-

step regression for these equations increased by around 2 stem/ha from the all-possible interval method.

Despite a small increase in SEE (around 1 stem/ha) for Equation 11 of the two-step regression for *Acacia crassicarpa* when compared with the two other approaches, the bias for this equation was smaller. This approach had bias around 0.5 stems/ha and 5 stems/ha smaller than the all-possible interval and longest interval approaches respectively for Equation 11. In addition, bias from Equation 22 of the two-step regression was 2 stems/ha smaller than the all-possible interval method.

Equation 4 of the two-step regression performed the best for *Eucalyptus pellita*. The bias of this equation was close to zero and it reduced significantly to about 14 stems/ha compared with the all-possible interval approach. Although the SEE increased by around 0.7 stems/ha, the MAE reduced by about 2.3 stems/ha.

Based on statistic values from Tables 33–35, most of the models of two-step regression had a smaller bias (MRES) compared with the other two approaches for all species. It was clear that the longest intervals performed the worse among those approaches for both *Acacia mangium* and *Eucalyptus pellita*. Meanwhile, the performances of all approaches were identical for *Acacia crassicarpa*.

Table 33 *Statistical values of validation for three best equations of different mortality approach for* Acacia mangium *using the same dataset.*

Equation	Author/Name	Statistical values of validation		
		MRES	SEE	MAE
	*All-possible intervals**			
2	Lundqvist–Korf (Schumacher Anamorphic II)	-6.818 (2)	139.4 (3)	94.9 (2)
22	Hossfeld IV	-7.293 (3)	139.4 (2)	94.7 (1)
30	Exponential decay anamorphic	-4.444 (1)	137.2 (1)	95 (3)
	*One-year interval (two-step regression)***			
2	Lundqvist–Korf (Schumacher Anamorphic II)	-1.912 (1)	137.3 (2)	96.4 (2)
8	von Bertalanffy–Richards Anamorphic	-2.759 (3)	137.1 (1)	96.2 (1)
30	Exponential decay anamorphic	2.754 (2)	137.8 (3)	96.7 (3)
	Longest interval			
16	Gompertz Polymorphic I	36.083 (3)	143.6 (1)	110.5 (3)
27	Anamorphic III	-8.478 (2)	146.3 (2)	108.6 (1)
28	Anamorphic IV	-8.374 (1)	146.8 (3)	109 (2)

Note: Ranking is shown in bracket. This ranking only compared the best three models.
**=same values as Appendix C. **= after adjustment using a probability function*

Table 34 *Statistical values of validation for three best equations of different mortality approaches for* Acacia crassicarpa *using the same dataset.*

Equation	Author/Name	Statistical values of validation		
		MRES	SEE	MAE
	*All-possible intervals**			
11	Weibull	-7.746 (1)	204.8 (1)	156.9 (1)
22	Hossfeld IV	-8.329 (2)	205.86 (2)	157.7 (2)
30	Exponential decay anamorphic	-9.241 (3)	205.9 (3)	158.2 (3)
	*One-year interval (two-step regression)***			
2	Lundqvist–Korf (Schumacher Anamorphic II)	-10.057 (3)	208.8 (3)	159 (3)
11	Weibull	-7.114 (2)	205.4 (1)	156.9 (1)
22	Hossfeld IV	-6.757 (1)	207.5 (2)	158.3 (2)
	Longest interval			
11	Weibull	-12.484 (1)	203.7 (1)	156.7 (1)
20	Hossfeld Anamorphic	-30.225 (3)	209.5 (3)	162.7 (3)
30	Exponential decay anamorphic	-13.017 (2)	206.2 (2)	158.2 (2)

Note: Ranking is shown in bracket. This ranking only compared the best three models.
**=same values with Appendix D. **= after adjustment using a probability function*

Table 35 *Statistical values of validation for three best equations of different mortality approaches for* Eucalyptus pellita *using the same dataset.*

Equation	Author/Name	Statistical values of validation		
		MRES	SEE	MAE
	*All-possible intervals**			
4	Schumacher/ Polymorphic II	14.526 (3)	184.80 (1)	141.0 (1)
17	Logistic I	13.205 (2)	186.14 (3)	142.5 (2)
27	Anamorphic III	12.608 (1)	186.06 (2)	142.9 (3)
	*One-year interval (two-step regression)***			
4	Schumacher/ Polymorphic II	0.445 (1)	185.5 (2)	138.7 (1)
15	Gompertz	9.71 (3)	184.6 (1)	139.6 (3)
22	Hossfeld Polymorphic	-0.75 (2)	185.5 (3)	139.1 (2)
	Longest interval			
27	Anamorphic III	15.336 (2)	186.4 (1)	143.7 (1)
28	Anamorphic IV	14.836 (1)	187.1 (2)	144.5 (3)
30	Exponential decay anamorphic	22.906 (3)	190.3 (3)	144 (2)

Note: Ranking is shown in bracket. This ranking only compared the best three models.
**=same values with Appendix E. **= after adjustment using a probability function*

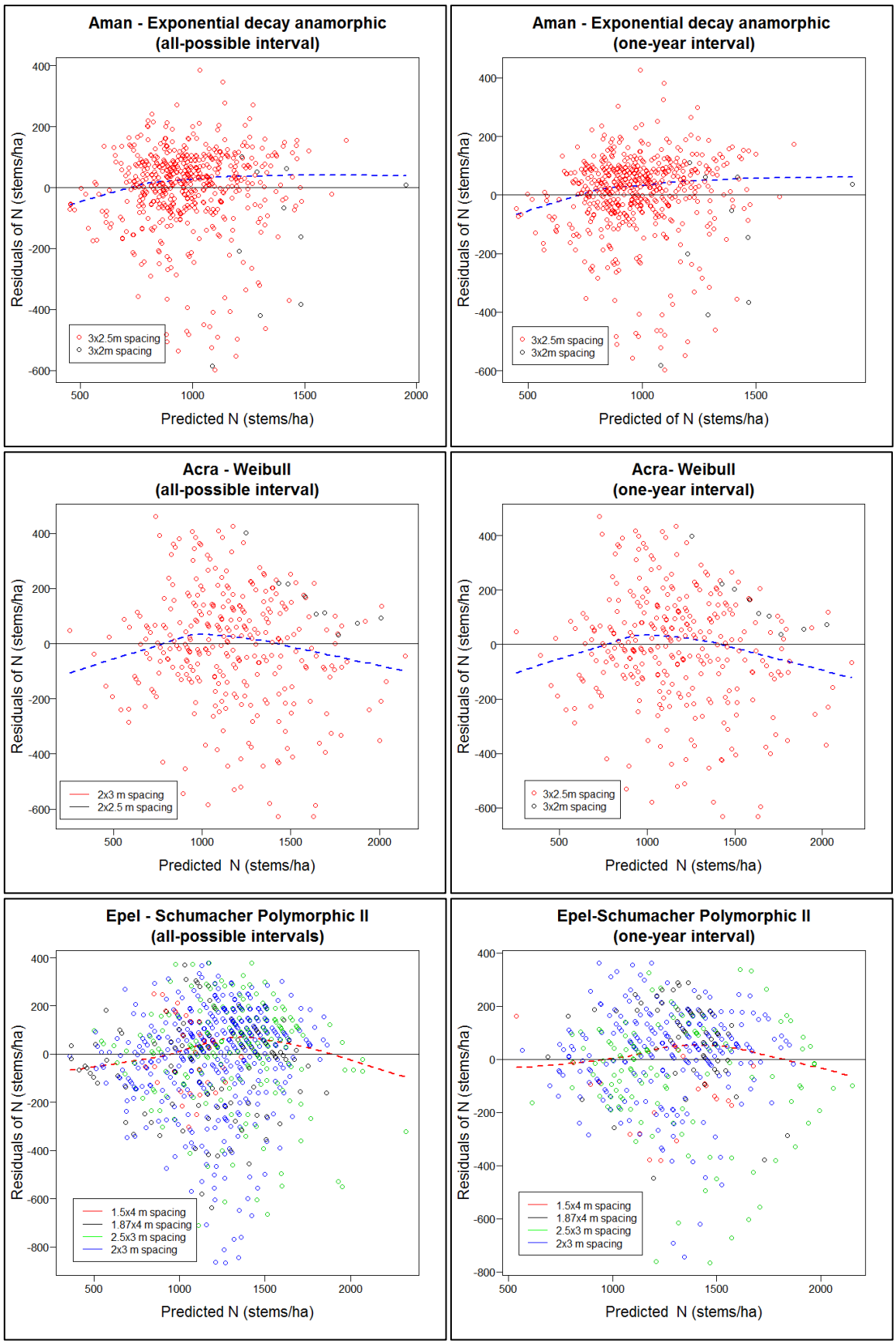

Figure 40 Comparison of residual plots between all-possible interval approach (left) and two-step regression (right) using the same validation dataset for *Acacia mangium* (top), *Acacia crassicarpa* (middle) and *Eucalyptus pellita* (bottom).

There was at least one model that appeared in both the all-possible interval and two-step regression approaches for all species. We then tried comparing the residual plots from these equations. We used Equation 30 for *Acacia mangium*, Equation 11 for *Acacia crassicarpa* and Equation 4 for *Eucalyptus pellita*. Figure 40 showed that the residual plots for both approaches were similar for all species. Based on the consideration above, we decided to use two-step regression as a method to predict mortality for these species.

5.3.2. Two-step Regression

Table 36 *Statistical values of mortality function using two-step regression approach*

Equation	Author/Name	Model fitting				Validation		
		MRES	SEE	MAE	AIC	MRES	SEE	MAE
		Acacia mangium						
2	Lundqvist–Korf (Schumacher Anamorphic II)	-3.351 (3)	113.6 (2)	80.3 (3)	14134 (3)	-7.308 (2)	118.5 (2)	80.63 (2)
8	von Bertalanffy–Richards Anamorphic	-3.342 (2)	113.4 (1)	80.1 (2)	14134 (2)	-7.33 (3)	118.3 (1)	80.63 (3)
30	Exponential decay anamorphic	-2.675 (1)	113.8 (3)	79.7 (1)	14132 (1)	-6.832 (1)	119 (3)	80.51 (1)
		Acacia crassicarpa						
2	Lundqvist–Korf (Schumacher Anamorphic II)	-2.061 (3)	142.6 (3)	109.6 (3)	7891 (3)	-15.315 (3)	158.6 (3)	123.7 (3)
11	Weibull	-1.764 (2)	139 (1)	107.4 (1)	7890 (1)	-14.606 (2)	156.2 (1)	121.9 (1)
22	Hossfeld IV	-0.943 (1)	141 (2)	108.6 (2)	7890 (1)	-13.951 (1)	157.5 (2)	123 (2)
		Eucalyptus pellita						
4	Schumacher/ Polymorphic II	-8.336 (2)	134 (1)	98.2 (1)	9276 (3)	1.623 (2)	136 (2)	98.8 (2)
15	Gompertz	-7.428 (1)	135.1 (3)	100.6 (3)	9272 (1)	4.838 (3)	135.1 (1)	98.4 (1)
21	Hossfeld Polymorphic	-9.336 (3)	134.3 (2)	98.7 (2)	9272 (1)	1.269 (1)	136.2 (3)	99.2 (3)

Note: Ranking is shown in bracket. This ranking only compared the best three models. These were the results after adjustment using a probability function

The statistical values of the two-step regression approach for mortality function are shown in Table 36. It was clear that Equation 30 for *Acacia mangium* and Equation 11 for *Acacia crassicarpa* performed the best, both in model fitting and during the validation process. Although the SEE value of Equation 30 was the worst, the difference was minimal (0.2–0.4). On the other hand, bias of Equation 11 for *Acacia crassicarpa* was larger by about 1 stem/ha than Equation 22, which had the smallest value of bias.

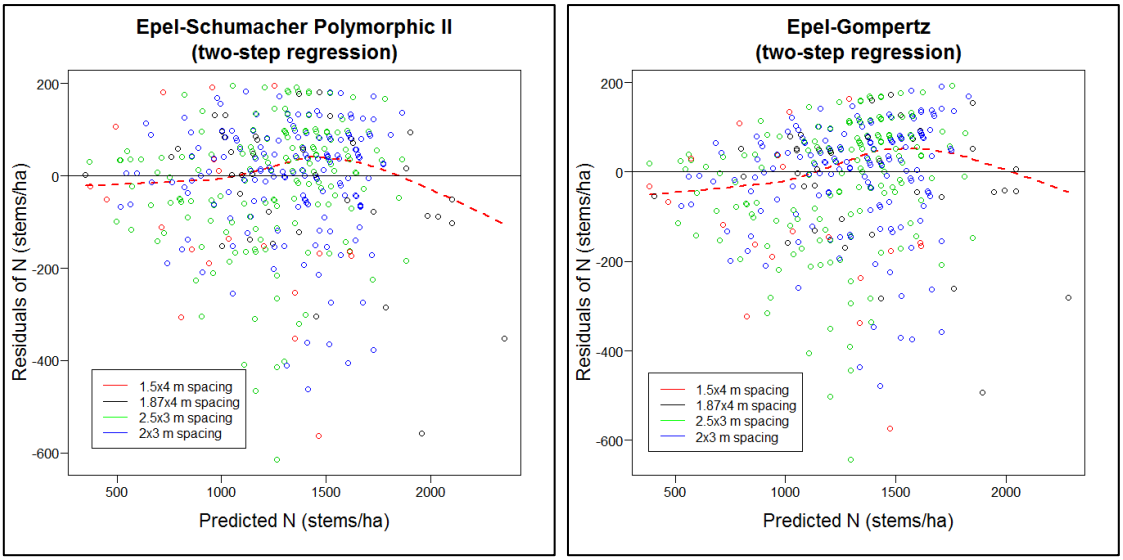

Figure 41 Comparison of residuals between Equation 4 (left) and Equation 15 (right) for *Eucalyptus pellita* in model construction after adjustment using a probability function.

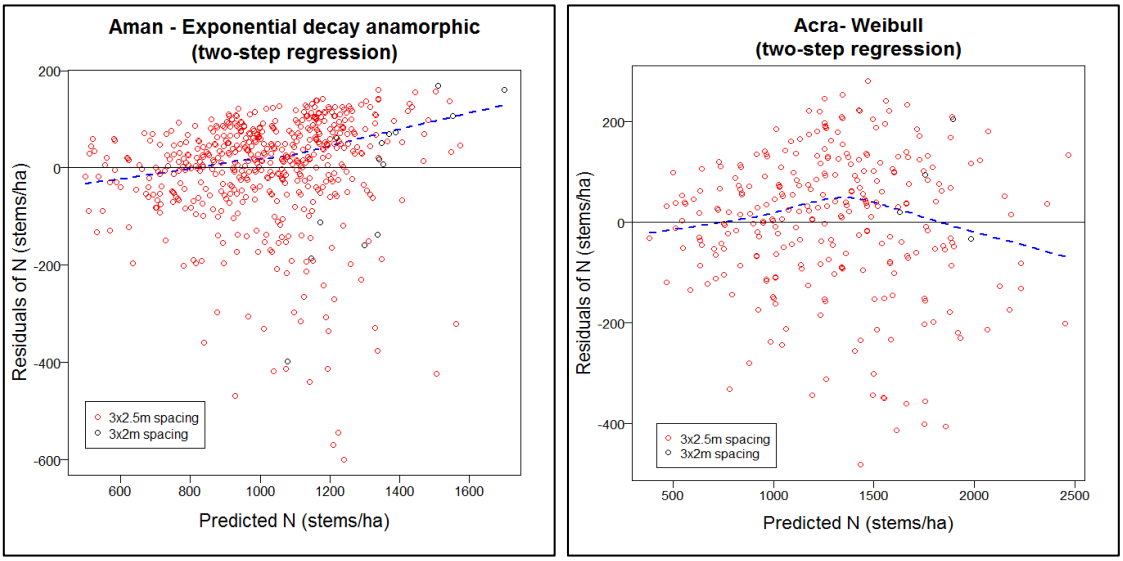

Figure 42 Residual plots of Equation 30 for *Acacia mangium (left)* and Equation 11 for *Acacia crassicarpa* (right) in model construction after adjustment using a probability function.

It was a little hard to determine the best mortality function for *Eucalyptus pellita*. However, we decided to choose Equation 4 based on the smallest SEE and MAE values in model fitting. The bias of this equation had a greater value by about 1 stem/ha in model fitting compared with Equation 15. The residual plots of these equations were identical, but Equation 4 had a slightly better residual (Figure 41).

We used starting values of MTH1 = 5.4 m, quadratic mean diameter (Dq1) = 5.8 cm and different stocking (N1) = 1667, 1333 and 1000 stems/ha at T1 = 1 year to see the projection of stocking until age 6 years for *Acacia mangium* against the actual data of mortality. Moreover, we used stocking (N1) = 2400, 1800 and 1200 stems/ha and stocking (N1) = 1900, 1500 and

1100 stems/ha at T1= 1 year to predict stocking until age 5 years for *Acacia crassicarpa* and *Eucalyptus pellita* respectively. These are shown in Figures 43–45 (left-hand side).

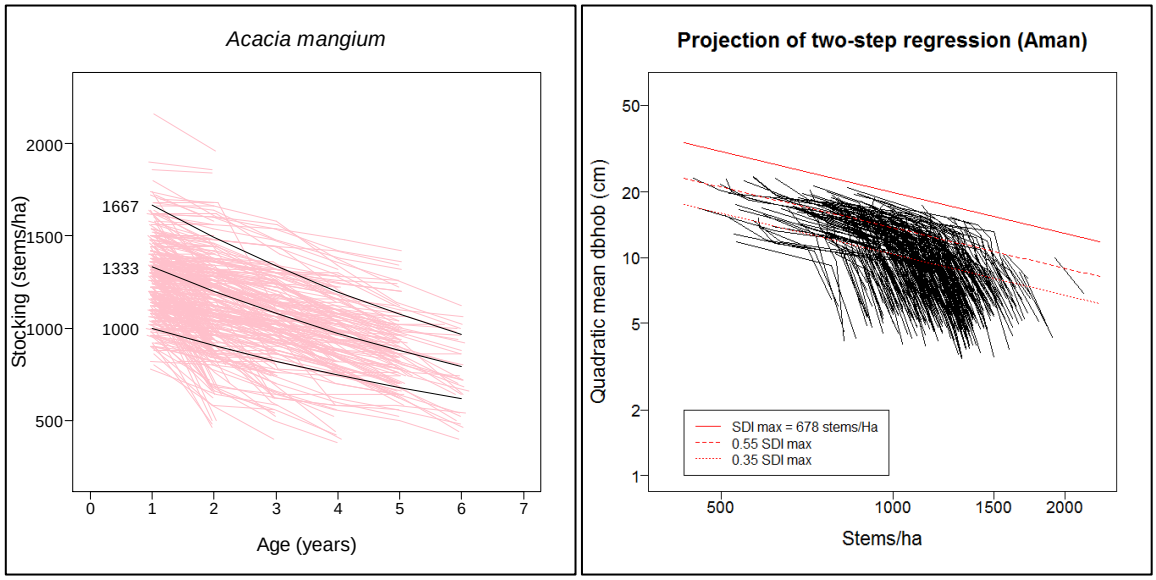

Figure 43 Equation 30 when fitted to the actual dataset using stocking values (1667, 1333 and 1000 stems//ha) at age one year (left) and the simulation of Equation 30 against quadratic mean dbhob using the first measurement of the actual dataset within a stand-density index (Reineke, 1933) framework (right) for *Acacia mangium*

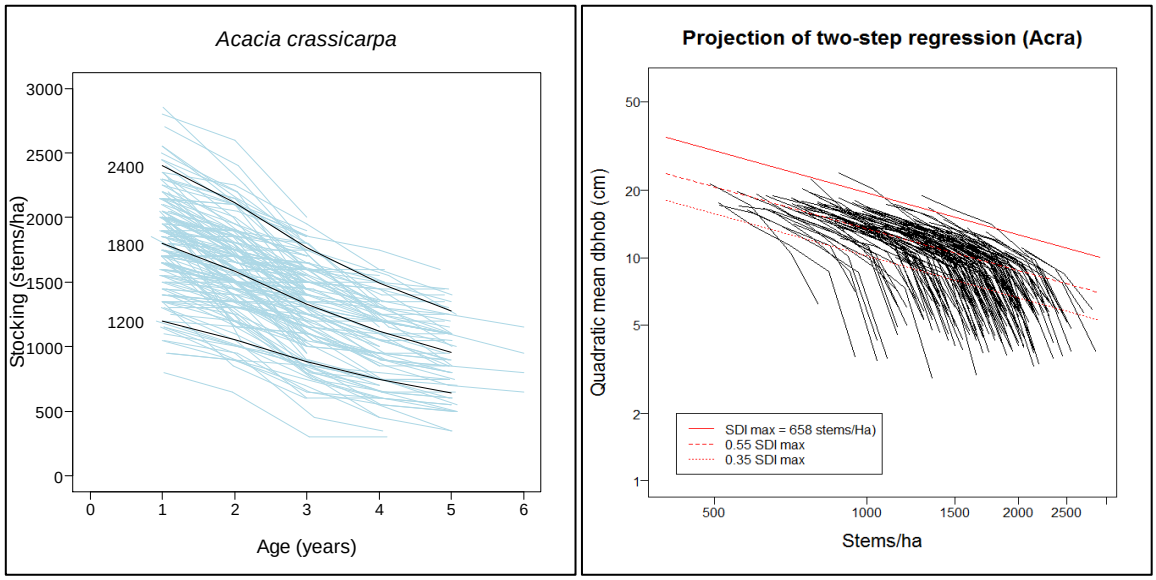

Figure 44 Equation 11 when fitted to the actual dataset using stocking values (2400, 1800 and 1200 stems//ha) at age one year (left) and the simulation of Equation 11 against quadratic mean dbhob using the first measurement of the actual dataset within a stand-density index (Reineke, 1933) framework (right) for *Acacia crassicarpa*.

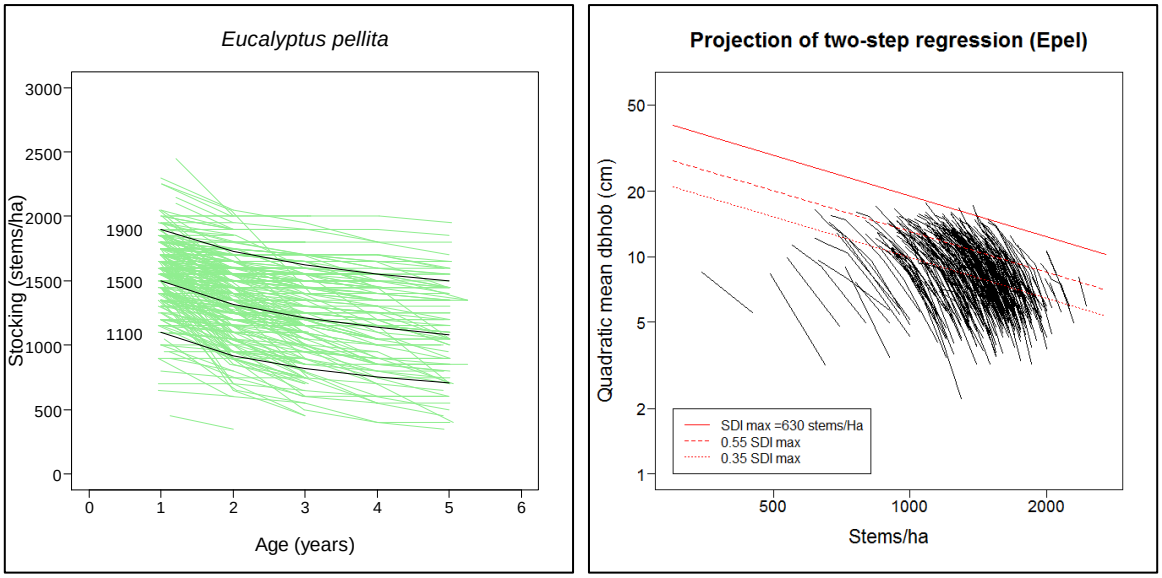

Figure 45 Equation 4 when fitted to the actual dataset using stocking values (1900, 1500 and 1100 stems//ha) at age one year (left) and the simulation of Equation 4 against quadratic mean dbhob using the first measurement of the actual dataset (Reineke, 1933) framework (right) for *Eucalyptus pellita.*

We also simulated mortality projections against quadratic mean diameter by using the first measurement values as the starting values. Using the equation of Reineke (1933) for stand-density index, we found that the maximum stand-density indices were 678 stems/ha, 658 stems/ha and 630 stems/ha for *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita* respectively. These simulations are also shown in Figures 43–45 (right-hand side).

5.4. Stand Volume (V)

We generated stand-volume equations using basal area (G) and mean top height (MTH) as independent variables as expressed in Equations 2.8–2.10. We found that some parameters of Equation 2.9 were insignificant for *Acacia mangium* and *Eucalyptus pellita*. For *Acacia crassicarpa*, the differences between Equations 2.8 and 2.9 were small. The SEE of Equation 2.8 were only 0.17 m^3/ha and MAE was only 0.2 m^3/ha bigger than Equation 2.9. Moreover, in term of validation, Equation 2.8 performed better than Equation 2.9 (Table 38) for all species. The residual plots of Equation 2.10 were biased for all species. Meanwhile, residual plots of Equations 2.8 and 2.9 were similar. Based on these considerations, we chose Equation 2.8 for projecting the stand volume for all species.

Due to the heteroscedasticity that appeared in all species, we needed to carry out either a transformation or weighted least squares regression meet the constant variance assumption. We tried to use 1/G, $1/G^2$ and 1/GH for the weighted least squares method. The weighted least

squares residual plots were similar and still exhibited a heteroscedastic pattern. An example of weighted least squares using 1/G as weight factor is also shown in Figures 46–48 (top right side). Log and scaled power transformations did not give better residuals and resulted in bias after the back-transformation (Figures 46–48). Moreover, a comparison of standard errors between the original Equation 2.8 and its modifications (weighted least squares, log and scaled power transformation) showed that the original equation gave the smallest SEE for all species (Table 39). Based on all these considerations, we chose the original equation for stand-volume prediction to be applied to all species.

Table 37 *Statistical values of model fitting for stand volume*

Equation	Coefficients				Model fitting			
	intercept	**G**	**MTH**	**GH****	**MRES**	**SEE**	**MAE**	**AIC**
				Acacia mangium				
2.8		1.025546		0.375828	-0.058 (3)	2.087 (2)	1.217 (1)	2503 (2)
2.9	0.443487*	1.116318	-0.158012	0.378778	0 (1)	2.07 (1)	1.227 (2)	2496 (1)
2.10	3.540735			0.422793	0 (2)	2.743 (3)	1.958 (3)	2821 (3)
				Acacia crassicarpa				
2.8		0.475767		0.371822	0.037 (3)	7.22 (2)	4.522 (2)	2823 (2)
2.9	-5.2756	0.54683	0.72458	0.34195	0 (1)	7.048 (1)	4.334 (1)	2805 (1)
2.10	2.054578			0.389254	0 (2)	7.256 (3)	4.626 (3)	2827 (3)
				Eucalyptus pellita				
2.8		0.724127		0.42551	-0.286 (3)	3.288 (2)	2.212 (2)	3130 (2)
2.9*	-2.62748	1.303564	-0.0034*	0.406455	0 (1)	3.175 (1)	2.057 (1)	3090 (1)
2.10	1.771742			0.454684	0 (2)	3.592 (3)	2.484 (3)	3236 (3)

*Note: Ranking is shown in the bracket, * = insignificant parameter, **= G multiple by MTH.*

Table 38 *Statistical values of validation for stand volume*

Equation	*Acacia mangium*			*Acacia crassicarpa*			*Eucalyptus pellita*		
	MRES	**SEE**	**MAE**	**MRES**	**SEE**	**MAE**	**MRES**	**SEE**	**MAE**
2.8	-0.167 (3)	2.205 (1)	1.183 (1)	1.429 (1)	7.333 (1)	4.596 (1)	-0.602 (3)	3.41 (2)	2.363 (2)
2.9*	-0.114 (2)	2.266 (2)	1.234 (2)	1.533 (3)	7.617 (3)	4.666 (2)	-0.333 (2)	3.264 (1)	2.169 (1)
2.10	-0.074 (1)	2.957 (3)	1.903 (3)	1.511 (2)	7.5 (2)	4.82 (3)	-0.312 (1)	3.674 (3)	2.562 (3)

*Note: Ranking is shown in the bracket, * = insignificant for Acacia mangium and Eucalyptus pellita*

Table 39 *Comparison of standard error between original equation and its modification*

SEE	*Acacia mangium*	*Acacia crassicarpa*	*Eucalyptus pellita*
Equation 2.8 (original)	2.087	7.22	3.288
Weighted least squares	2.088	7.249	3.327
Log transformation	4.509	12.205	5.447
Scaled power transformation	2.649	7.292	4.309

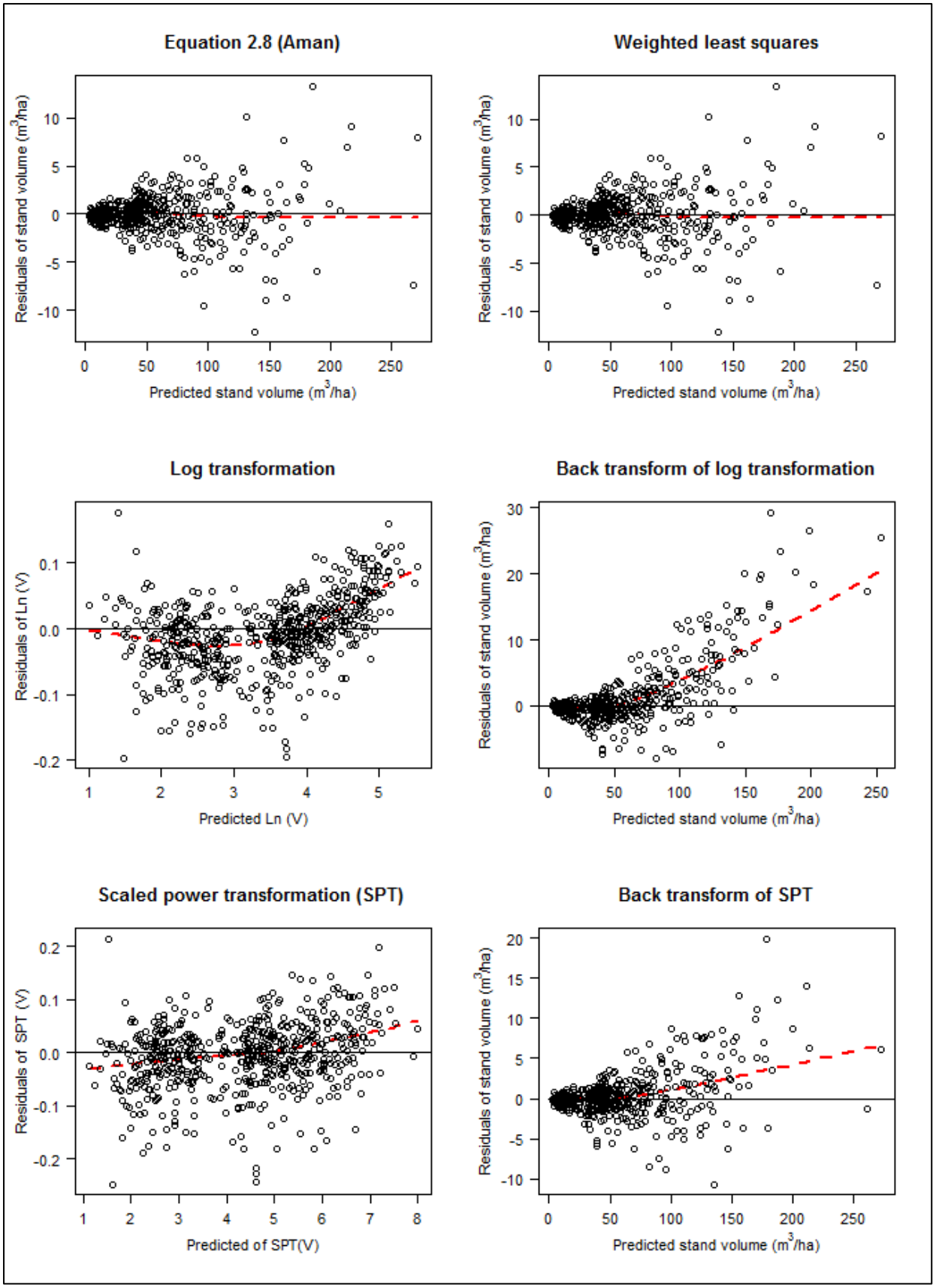

Figure 46 Residual plots of Equation 2.8 (top left), weighted least squares with weight factor = 1/G (top right), log transformation (middle left), back transform of log transformation (middle right), scaled power transformation (bottom left) and back transform of scale power transformation (bottom right) for *Acacia mangium*

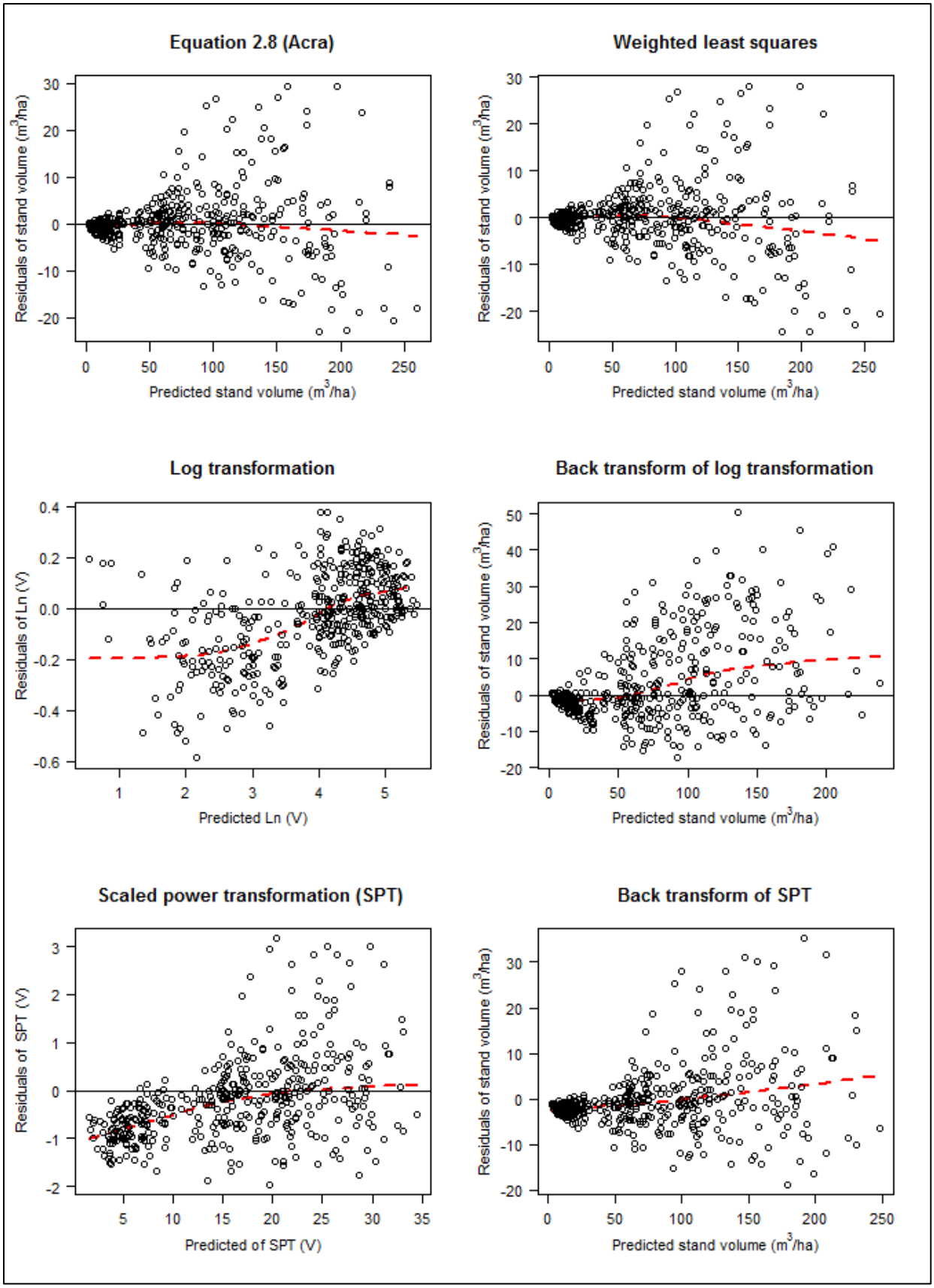

Figure 47 Residual plots of Equation 2.8 (top left), weighted least squares with weight factor = 1/G (top right), log transformation (middle left), back transform of log transformation (middle right), scaled power transformation (bottom left) and back transform of scale power transformation (bottom right) for *Acacia crassicarpa*

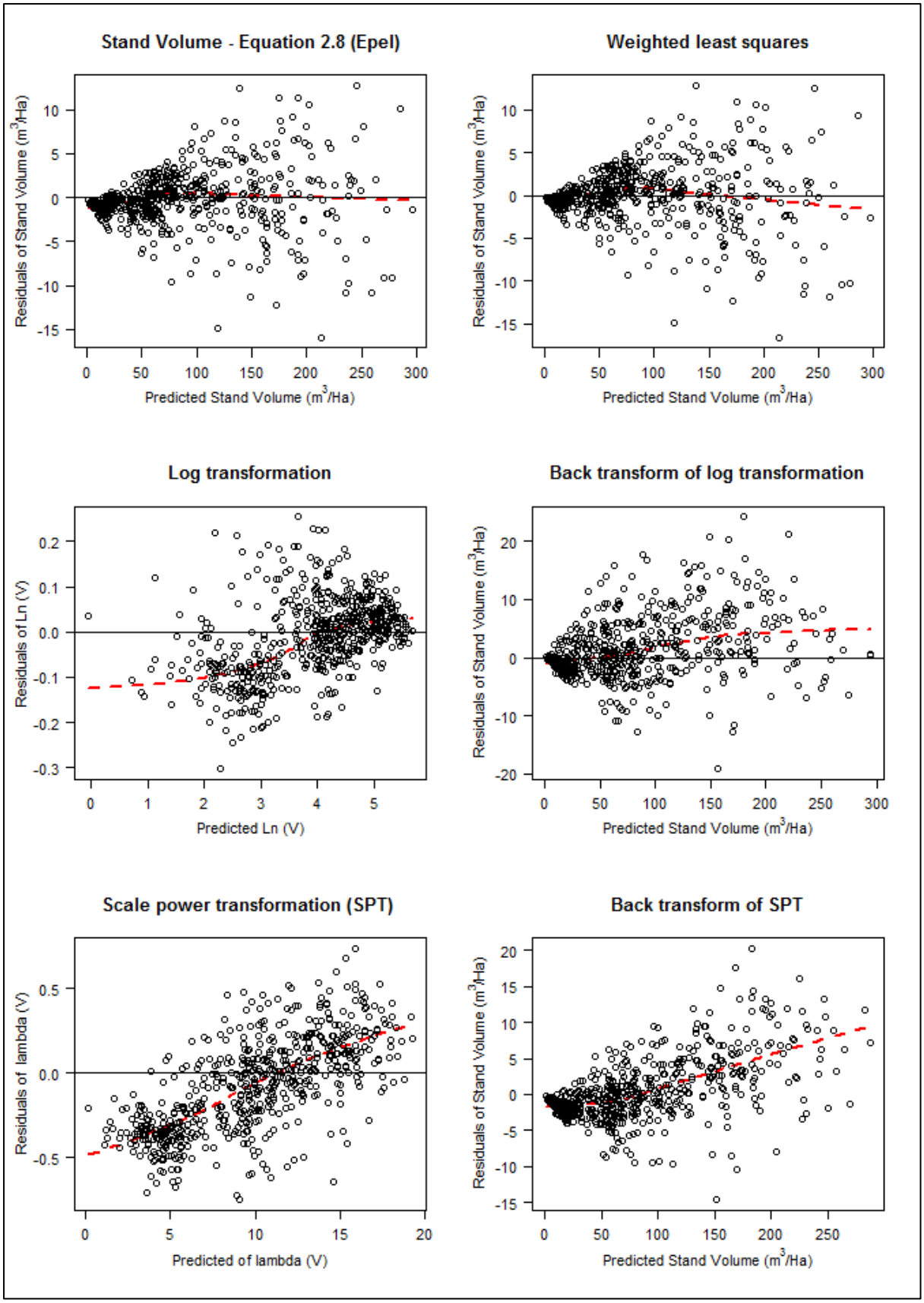

Figure 48 Residual plots of Equation 2.8 (top left), weighted least squares with weight factor = 1/G (top right), log transformation (middle left), back transform of log transformation (middle right), scaled power transformation (bottom left) and back transform of scale power transformation (bottom right) for *Eucalyptus pellita*

5.5. Augmented Model

Many factors can influence the growth of a tree. During this study we examined the influence of silvicultural treatments and climatic factors on growth of the three species. Overall, pruning and thinning were not applied to this study area, and the singling used for the *Acacia mangium* plantations was not recorded well. The things that we may inspect are the spacing treatment and the seedling source.

Several studies found that the provenance (Jones, Hayes, & Hamilton, 2001; Nagamitsu, Nagasaka, Yoshimaru, & Tsumura, 2014) and some extreme spacing (Hein, Weiskittel, & Kohnle, 2008; Miah, Islam, Habib, & Moula, 2014) had significant influences on growth. For instance, the New Guinea provenance of *Eucalyptus pellita* was superior to the Queensland provenance in early growth in northern Australia and Colombia (Harwood, Alloysius, Pomroy, Robson, & Haines, 1997; Nieto, Giraldo-Charria, Sarmiento, & Borralho, 2016).

However, since the PSPs for these species were not designed for experimental work, the datasets were not balanced for both spacing and provenance. Based on an interview in the field, the selection of provenance mostly depends on the seed availability for these species. There are too many categories of the seedling type for these species and one plot may consist of more than one provenance. On the other hand, the spacing we can see in the residual plots in Chapter 5.1 to 5.3 seems unlikely to have some kind of pattern that describe the impact of spacing on growth.

Table 40 *Correlations between weather, site index and elevation for each type of soil data*

Soil	**Alfisol**		**Histosol**		**Inceptisol**		**Ultisol**	
Acacia mangium	**R**	**T**	**R**	**T**	**R**	**T**	**R**	**T**
Elevation	-0.242	0.242	-	-	-0.117	0.082	-0.226	0.21
Site Index	0.181	-0.181	-	-	-0.146	0.152	-0.213	0.214
Acacia crassicarpa	**R**	**T**	**R**	**T**	**R**	**T**	**R**	**T**
Elevation	-	-	-0.031	-0.374	-0.201	-0.316	0.245	-0.86
Site Index	-	-	0.227	0.212	-0.38	-0.289	-0.25	0.424
Eucalyptus pellita	**R**	**T**	**R**	**T**	**R**	**T**	**R**	**T**
Elevation	-	-	0.799	0	-0.53	-0.361	-0.304	-0.522
Site Index	-		-0.19	0	-0.309	-0.116	0.129	0.042

Note: R = mean annual precipitation, T = mean annual temperature

Edaphic data are available for all species on order level and subgroup level based on United States Department of Agriculture (USDA) Soil Classification. As we can see in Table 40, an increase in elevation corresponded to a reduction in temperature for *Acacia crassicarpa* and

Eucalyptus pellita. On the other hand, an increase in elevation also increased the temperature for *Acacia mangium*. Rainfall decreased as the altitude increased, and this pattern was dominant for all species. However, the correlation between climate data and elevation was not high (less than 0.6) for all soil type.

Figure 50 shows that the difference in number of plots for each soil type was high for *Acacia crassicarpa* and *Eucalyptus pellita*. Although the proportion of inceptisol and ultisol was almost the same, there was a high number of missing soil data for the remaining PSP of *Acacia mangium*. Regardless of the missing soil data, we analysed the influence of soil type using a least means square (lsmeans) method for MTH, basal area and stocking at age 5 years, and found that *Acacia mangium*'s growth was not significantly affected by soil (Figure 49), and hence we removed the soil variable as a part of our analysis.

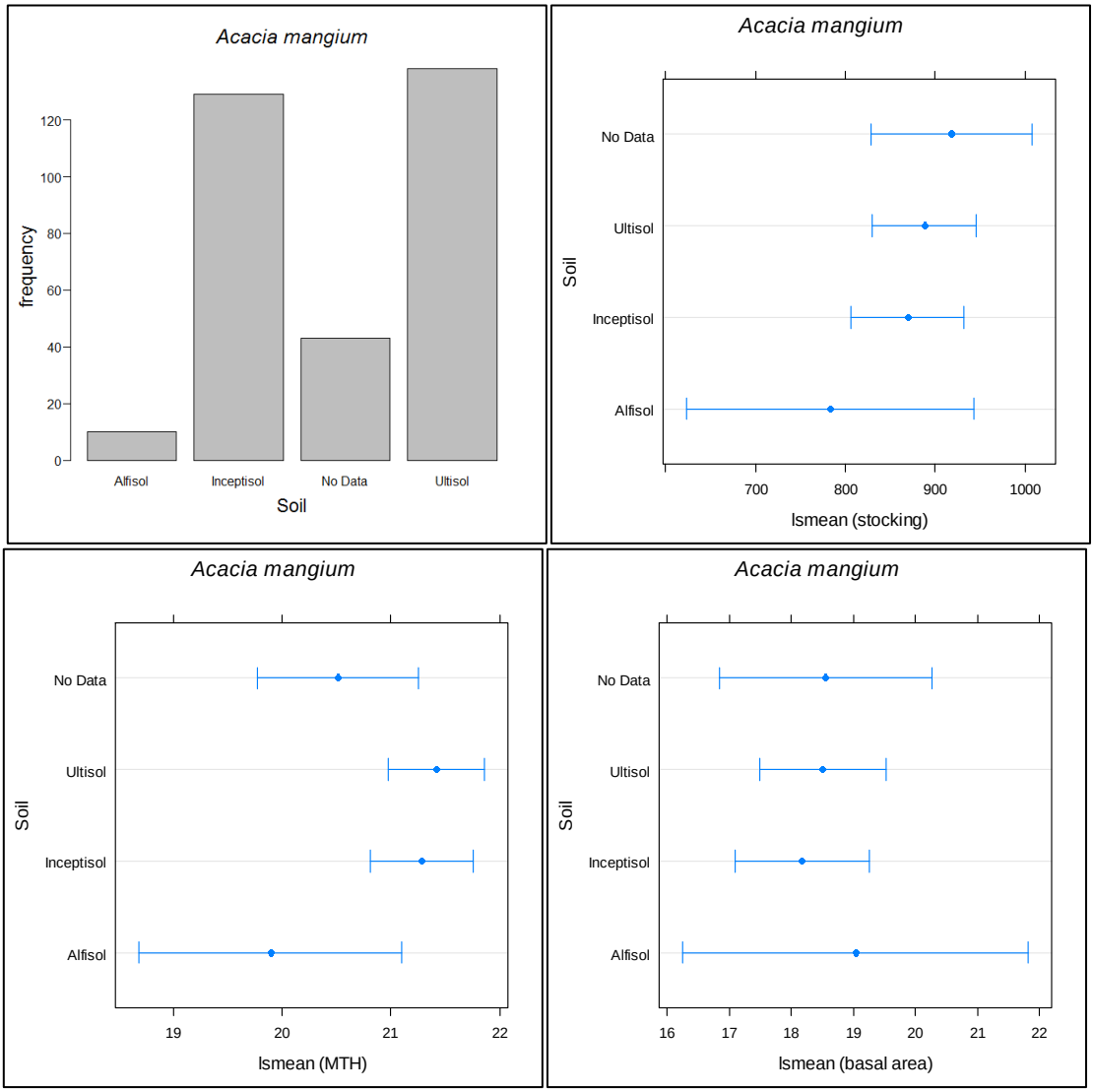

Figure 49 Soil type distribution (top left) and soil type effect on stocking (top right), MTH (bottom left) and basal area (bottom right) for *Acacia mangium*

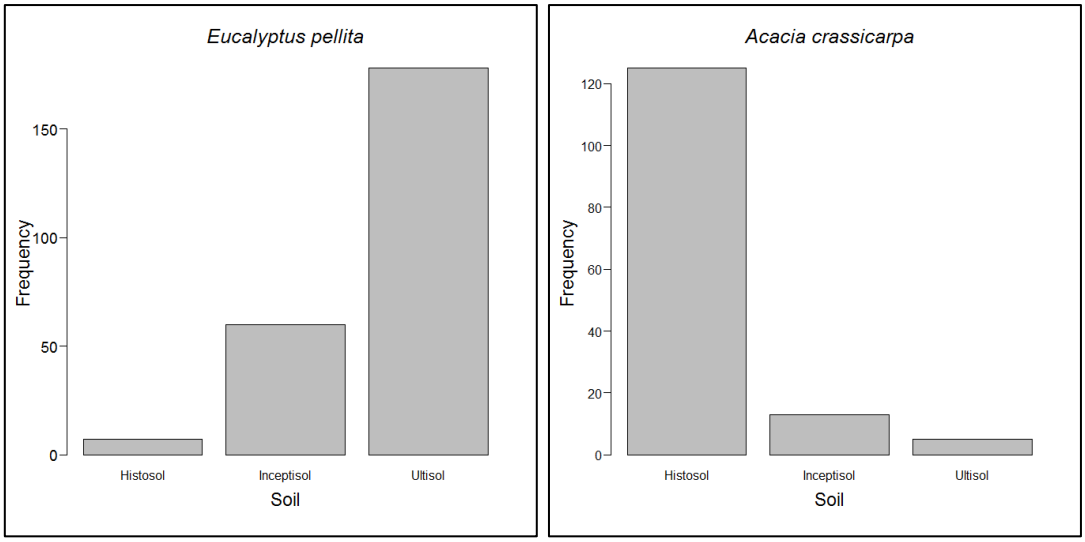

Figure 50 Soil type distribution for *Eucalyptus pellita (*left) and *Acacia crassicarpa* (right)

The remaining factors were elevation, mean annual precipitation and mean annual temperature. We then added these variables into parameters from the equation chosen from section V.1 to V.3. The *b* and *c* parameters then became:

$$b = b_0 + b_1X_1 + \cdots + b_nX_n$$
$$c = c_0 + c_1X_1 + \cdots + c_nX_n$$

where *X* could be rainfall, elevation or temperature variables. The smallest standard error of the augmented equations obtained by adding these variables can be seen in Table 41. The details of each combination when adding these variables into the original equation can be seen in Appendix F.

Table 41 *Standard error of augmented equations (with additional elevation, rainfall or temperature)*

Variable	SEE			Note	
	Original equation	Augmented equation	Reduction	Variable added	into parameters
			Acacia mangium		
MTH	1.315	1.297	1.37 %	Elevation	b and c
G	2.212	2.14	3.25%	Elevation	c
N	112.8	110.2	2.3 %	Elevation	b and c
			Acacia crassicarpa		
MTH	1.363	1.357	0.44 %	Elevation	b and c
G	2.35	2.242	4.5 %	Rainfall	b
N	134.9	134.3[#]	0.04 %	Temperature	b and c
			Eucalyptus pellita		
MTH	2.013	1.991	1.09%	Elevation	b and c
G	2.512	2.485	1.07%	Rainfall	b and c
N	135.4	132.8[#]	1.92 %	Elevation and temperature	b and c

Note: MTH = Mean top height, G = basal area, N = stocking before adjustment using probability function. [#]insignificant parameters, [##]significant at $\alpha = 0.01$ on parameter chcking process.

From Table 41, we can see that the mortality equation for *Acacia crassicarpa* and *Eucalyptus pellita* did not pass the parameter checking process using one random interval for each plot. We then removed these equations for simulation of the projection phase. Using the same starting value that we used previously in section V.3.2, the future yield of augmented models are shown in Figures 51–54.

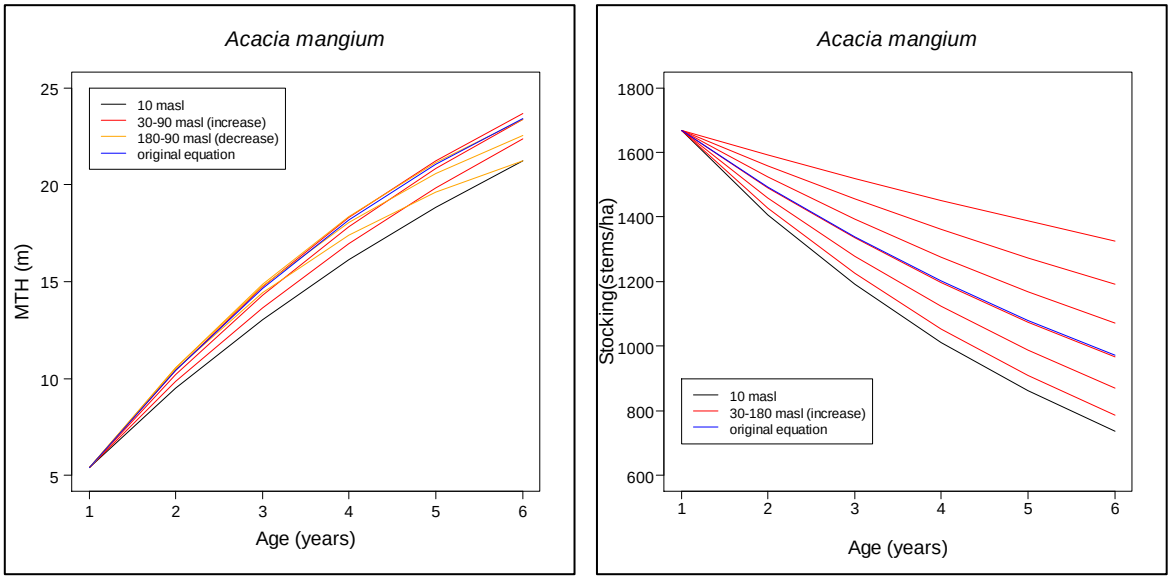

Figure 51 The projection of augmented equations of MTH (left) and mortality (right) for *Acacia mangium*

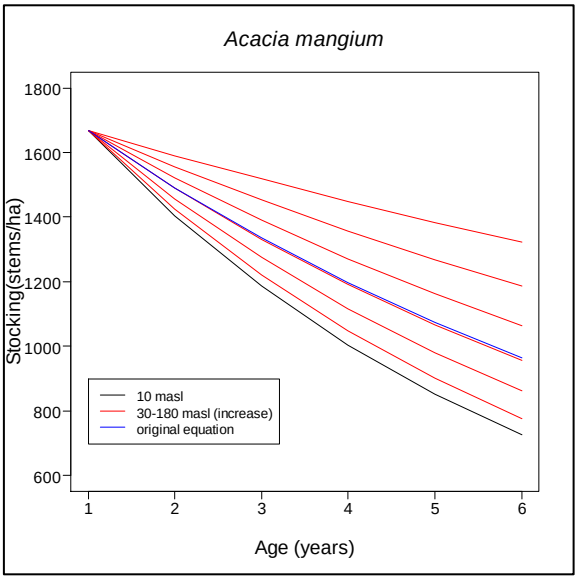

Figure 52 The projection of augmented equation of basal area for A*cacia mangium* as an effect of elevation

From those graphs, we can see that elevation had a great influence on *Acacia mangium*. The original equation gave a similar result when elevation was around 90 metres above sea level (masl) for MTH and stocking, and the basal area of the augmented equation gave the same

result as the original one when the elevation was around 100 metres. The overall result of the validation process gave better statistical values than the original model (Table 42). Although the values were small for standard error (SEE) and MAE, the bias gave a reduction of more than 50% for basal area.

The growth of MTH slightly increased as the elevation increased from 10 to 90 masl, and after 90 masl, MTH gradually decreased for *Acacia mangium*. Coomes and Allen (2007) found that elevation had a negative impact on tree growth for mountain beech in New Zealand, due to a reduction of intensity of light competition. In the FSS plantation, 24% of the total area had a slope higher than 25% (PT Fajar Surya Swadaya, 2011) and based on observation in the field, the higher elevation tended to have a steep contour.

Table 42 *The comparison of statistical values of validation between the original and augmented model*

Variable	Equation	MRES	SEE	MAE
		Acacia mangium		
MTH	Original	-0.006	1.471	1.081
	Augmented	0.006	1.470	1.079
G	Original	-0.112	2.491	1.911
	Augmented	-0.051	2.345	1.808
N	Original	-6.832	119.0	80.510
	Augmented	-5.968	115.3	77.259
		Acacia crassicarpa		
MTH	Original	- 0.327	1.450	1.178
	Augmented	- 0.346	1.441	1.164
G	Original	-0.041	2.474	1.917
	Augmented	-0.054	2.334	1.824
		Eucalyptus pellita		
MTH	Original	0.153	2.126	1.612
	Augmented	0.128	2.092	1.569
G	Original	0.217	2.512	1.815
	Augmented	0.148	2.406	1.816

Note: MTH = Mean top height, G = basal area, N = stocking after adjustment using probability function

The augmented model of basal area for *Acacia crassicarpa* as an effect of mean annual rainfall also gave a significant result. The difference of basal area can be up to 5 m^2/ha from two different sites that have differences of 500 mm/year of mean annual rainfall (Figure 53). Meanwhile, the differences of MTH was only around 0.7 metres from two different sites that have differences of 50 masl of elevation (Figure 53). The statistical values gave a better result for the standard error and MAE (Table 42) for both basal area and MTH.

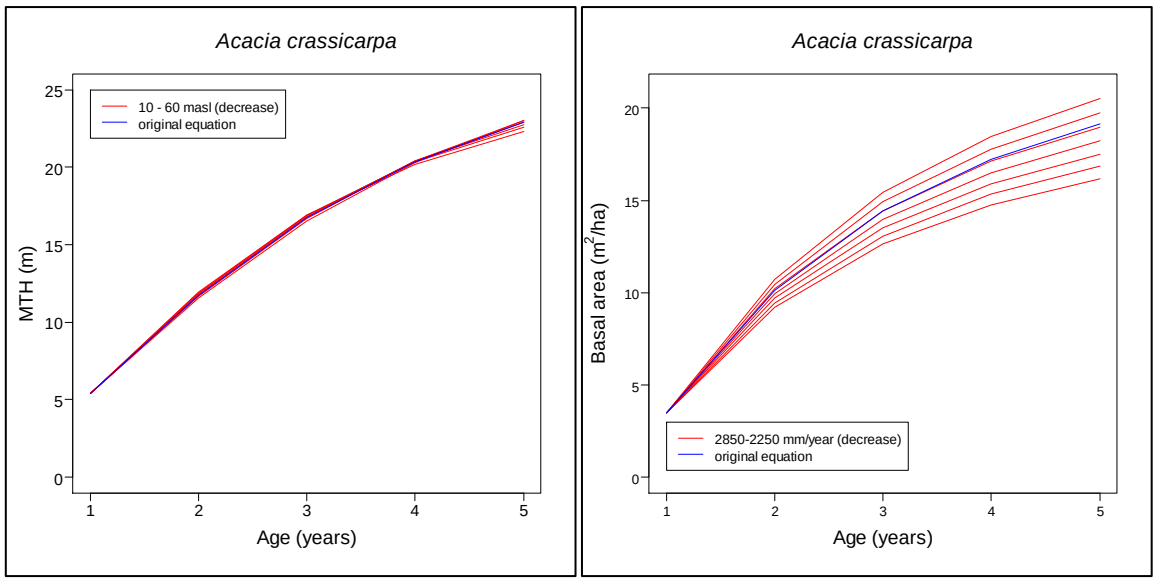

Figure 53 The projection of augmented equation of MTH as an effect of elevation (left) and basal area as an effect of mean annual rainfall (right) for *Acacia crassicarpa*

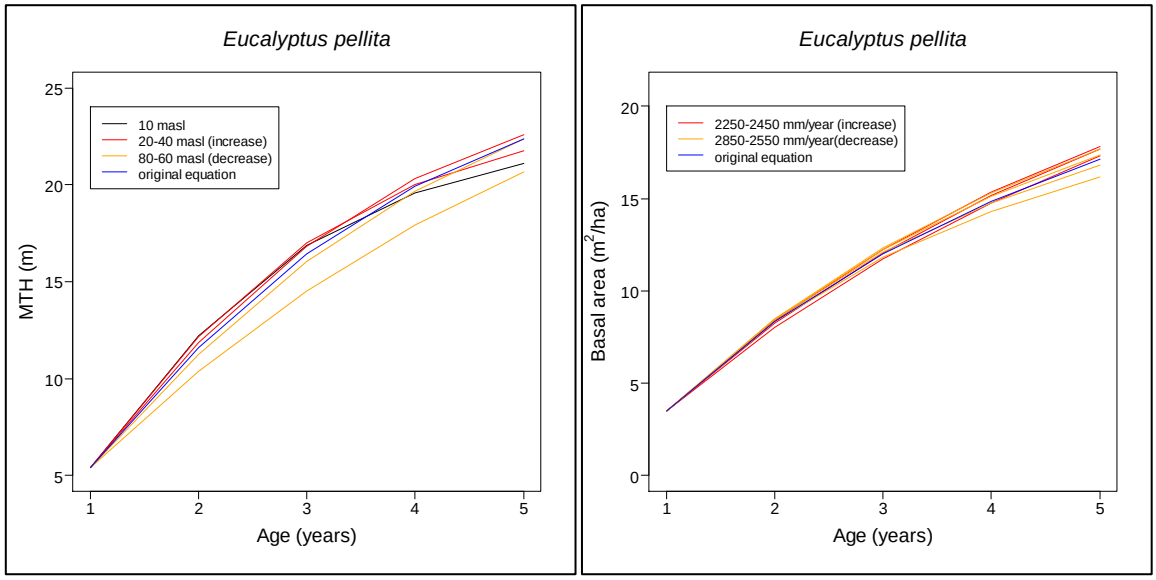

Figure 54 The projection of augmented equation of MTH as an effect of elevation (left) and basal area as an effect of mean annual rainfall (right) for *Eucalyptus pellita*

The effect of elevation on MTH and mean annual rainfall on basal area seemed small for *Eucalyptus pellita* (Figures 54). The change of slope and shape from the original models gave a skew pattern. The growth of MTH was slightly increased as the elevation increased from 10 to 40 masl, and MTH gradually decreased with elevations over 40 masl, The effect of mean annual rainfall on basal area also had this pattern with the turning point around 2450 mm/year. Schippers, Sterck, Vlam, and Zuidema (2015) tested the effect of rainfall on stem growth of *Toona ciliate* found that rainfall had a positive impact on inter-annual stem growth in Thailand

monsoon forest. However, the statistical values of augmented model gave the best result compared to the original equation for this species.

One of the consideration of the model is the simplicity of the model and model adaptability. The use of an augmented equation – formed by adding a site variable in a model - depends on the users addressing the question "Will it be practical to apply in the field?" The augmented equations of MTH, basal area and mortality for *Acacia mangium* are written below:

MTH:

$$H_2 = (b_0 + b_1 * elevation)\left(\frac{H_1}{(b_0 + b_1 * elevation)}\right)^{\frac{\ln(1-e^{-(c_0+c_1*elevation)\,T_2})}{\ln(1-e^{-(c_0+c_1*elevation)T_1})}}$$

where: $b_0 = 43.3479068$, $b_1 = -0.0991418$, $c_0 = 0.0882680$, $c_1 = 0.0013318$

Basal area:

$$G_2 = e^{\ln(G_1)\left(\frac{T_1}{T_2}\right)^{(c_0+c_1*(elevation))}+b\left(1-\left(\frac{T_1}{T_2}\right)^{(c_0+c_1*(elevation))}\right)}$$

where: $b = 3.4677914$, $c_0 = 0.7156206$, $c_1 = 0.0021350$.

Mortality :

$$N_2 = N_1 e^{-(b_0+b_1*elevation)(T_2-T_1)}$$

Where: $b_0 = -0.1909446$, $b_1 = 0.0007847$.

The augmented equation of MTH and basal area for *Acacia crassicarpa* is written below:

MTH :

$$H_2 = (b_0 + b_1 * elevation)\left(\frac{H_1}{(b_0 + b_1 * elevation)}\right)^{\frac{\ln(1-e^{-(c_0+c_1*elevation)\,T_2})}{\ln(1-e^{-(c_0+c_1*elevation)T_1})}}$$

where: $b_0 = 30.019985$, $b_1 = -0.071118$, $c_0 = 0.349687$, $c_1 = 0.002520$

Basal area:

$$G_2 = e^{\ln(G_1)\left(\frac{T_1}{T_2}\right)^{c}+(b_0+b_1*Rainfall)\left(1-\left(\frac{T_1}{T_2}\right)^{c}\right)}$$

where $b_0 = 4.515$, $b_1 = -0.0004855$, $c = 1.051$.

The augmented equations of MTH and basal area for *Eucalyptus pellita* are written below.

MTH:

$$H_2 = (b_0 + b_1 * elevation)\left(\frac{H_1}{(b_0 + b_1 * elevation)}\right)^{\frac{\ln(1-e^{-(c_0+c_1*elevation)\,T_2})}{\ln(1-e^{-(c_0+c_1*elevation)T_1})}}$$

where $b_0 = 21.5169362$, $b_1 = 0.1292578$, $c_0 = 0.7038825$, $c_1 = -0.0059748$.

Basal area:

$$G_2 = e^{\ln(G_1)\left(\frac{T_1}{T_2}\right)^{(c_0+c_1*(Rainfall))}+(b_0+b_1*Rainfall)\left(1-\left(\frac{T_1}{T_2}\right)^{(c_0+c_1*(Rainfall))}\right)}$$

where $b_0 = 7.499$, $b_1 = -0.001404$, $c_0 = -0.5129$, $c_1 = 0.00043$.

5.6. Discussion and Conclusions

Empirical stand-level models for *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita* were created during this study. We found that the von Bertalanffy–Richards Polymorphic (Equation 9) was the best equation for MTH prediction for all species. The range of standard errors for this equation was around 1.3 metres for both species of *Acacia* and 2 metres for *Eucalyptus pellita*. The two-parameter Schumacher Polymorphic (Equation 4) also gave the best result compared with other equations for all species, with a standard error range of 2.2 to 2.5 m^2/ha. Several studies (Casnati, 2016; Methol, 2001; Torres Vélez & Del Valle, 2007) also found that the von Bertalanffy–Richards Polymorphic was the best model for MTH projections and the two-parameters of Schumacher Polymorphic was the best model for predicting basal area (Clutter & Jones, 1980; Methol, 2001; Zhao, 1999).

The mortality equation was hard to model. Methol (2001) found the longest interval was the best method for stocking projection compared to all-possible interval and nonoverlapping interval data for *Eucalyptus grandis* in Uruguay and *Pseudotsuga menziesii* in New Zealand. Although he recommended using the longest-interval method, he found that the two-step regression approach gave the lowest bias for *Pinus radiata* in New Zealand. He explained that the longest-interval method would provide a higher chance for mortality to occur and it can solve the problem described by Woollons (1998). However, Zhao (1999), who compared this method earlier for *Pinus radiata* in New Zealand, found that the all-possible-mixed interval approach was the best, but the advantage of this method was not statistically significant when compared with the other two approaches. Moreover, Lee (1998), who compared the mixed interval (all-possible growth interval), shortest interval and longest interval methods, found

that the mixed interval approach was the best for projecting the growth of Douglas fir in New Zealand. Casnati (2016) on the other hand, simply used a two-step regression method for generating mortality equations for *Pinus taeda* and *Eucalyptus grandis* in Uruguay. Based on the studies above, it was concluded that the use of growth intervals was dependent on the local dataset for each species and it could not be applied universally for other species (Zhao, 1999)

We compared three different approaches (all-possible, longest, and one-year growth interval) and chose the two-step regression (Woollons, 1998) by employing a one-year growth interval dataset and probability function for mortality projection. One of the considerations for selecting this approach was that the bias we got from the two-step regression showed a greater reduction than the other two approaches. Based on the result in section V.3.1 (Tables 30–32), the reduction of bias also depended on the model. For example, the exponential decay anamorphic equation gave a reduction in mean bias of about 98% and the Lundqvist–Korf equation gave a decrease of about 33% compared with the all-possible interval method for *Acacia mangium* (Table 33).

Each species has its own mortality equation for two-step regressions, unlike the MTH and basal area models. The exponential decay anamorphic (Equation 30) was the best for stocking projection for *Acacia mangium*, with standard error around 113 stems/ha. The Weibull anamorphic (Equation 11) was the best equation for *Acacia crassicarpa* with standard error around 139 stems/ha and the two-parameters of Schumacher Polymorphic (Equation 4) gave the best result for *Eucalyptus pellita* with standard error around 134 stems/ha. The limitation of mortality for *Acacia mangium* was the unrecorded singling treatment. This singling treatment definitely affected the mortality equation that was produced in this study.

Several variables were tested for inclusion in augmented models. However, due to unbalanced datasets, we could only use the mean annual temperature, mean annual rainfall and elevation as additional variables in augmented models. Reductions of standard error for *Acacia mangium* were 1.4% for MTH, 3.3% for basal area and 2.3% for mortality by adding an elevation variable. Elevation in the FSS area ranged from 4-406 masl (PT Fajar Surya Swadaya, 2011). However, the elevation of PSPs used in this study ranged from 10-184 masl (Table 3). Hence, the augmented models of *Acacia mangium* cannot be applied to a hilly area that has an elevation higher than 184 masl.

Acacia crassicarpa and *Eucalyptus pellita* had augmented models for MTH and basal area. The reduction of standard error for *Acacia crassicarpa* was 4.5% by adding a mean annual

rainfall variable for basal area and 0.4% for MTH by adding elevation into the original equations. The standard errors of augmented models for *Eucalyptus pellita* were small (around 1%) and obtained by adding elevation in the MTH equation and mean annual rainfall for basal area. For *Acacia crassicarpa* and *Eucalyptus pellita*, augmented mortality models did not have significant parameters in their equations. Therefore we do not recommend that the augmented model of mortality be applied to these species.

The use of augmented models had to be carefully applied. An additional input of elevation or climatic variable should address the feasibility of forest inventory practice in the field. We do not recommend that the augmented model of mortality be applied to *Acacia crassicarpa* and *Eucalyptus pellita*.

The best stand-volume equation was Equation 2.8, which employs basal area/ha (G) and (basal area × mean top height) (GH) as input variables. However, the limitation of this equation was the presence of heteroscedasticity on residual plots. Transformations (either log or scaled power) and weighted least-squares analysis did not improve the residuals or give a bigger standard error than the original model. Hence, the original equation was still used for the stand-volume projections for all species.

CHAPTER VI

Diameter Distribution Models

6.1. Introduction

Forest managers need information to govern their forests at different levels of resolution or stand to help them in the decision-making process (Frazier, 1981). While a stand-level model is simple (Munro, 1974) and individual tree models are complex (Burkhart & Tomé, 2012), the diameter distribution lies between these two levels of resolution (Husch et al., 2003; Nord-Larsen, 2006). A diameter distribution model is essential in estimating the range of product expected from a stand (Wang & Rennolls, 2005) and it is also helpful in forest evaluation and scheduling future silvicultural treatments (Nanos & Montero, 2002). Moreover, it directly affects the choice in harvesting stage activities such as the use of harvesting equipment (Lima, Bufalino, Alves Junior, Silva, & Ferreira, 2017)

A wide range of probability density functions are used in forestry for estimating the diameter distributions of stands, with three-parameter Weibull, four-parameter beta and S_B models being the most commonly used (Wang & Rennolls, 2005). In this study, we used the reverse Weibull distribution proposed by Kuru et al. (1992) by employing the maximum dbhob instead of minimum dbhob as a location parameter. The use of maximum dbhob minimalises the limitation that is commonly produced when using minimum dbhob (see section 2.4).

The objective of this study is to create the diameter distribution models for all species using the reverse three-parameter Weibull distribution. In this section, we also create generalised height–dbhob models as complementary analysis for validating the diameter distribution models.

6.2. Method

6.2.1. Reverse Weibull Distributions

We used the three-parameter reverse Weibull distribution (Equation 2.7) to generate the diameter distribution models for this study. The parameters *a*, *b* and *c* in that distribution's equation were calculated by using the procedure proposed by Garcia (1981). Variables of this equation are basal area/ha (*G*), stems/ha (*N*), maximum dbhob (Dmax) and the standard

deviation of dbhob (Dstd). The cumulative density function (*cdf)* and the probability density function (*pdf*) of the reverse of the three-parameter Weibull distribution are written below (Equations 2.6 and 2.7).

The probability density function (pdf):

$$f(X) = \frac{c}{b}\left(\frac{a-X}{b}\right)^{c-1} exp\left[-\left(\frac{a-X}{b}\right)^{c}\right], \qquad \text{(Equation 2.6)}$$

The cumulative density function (cdf):

$$F(X) = exp\left[-\left(\frac{a-X}{b}\right)^{c}\right] \quad \text{if } -\infty \le X \le a \qquad \text{(Equation 2.7)}$$

$$F(X) = 1, \quad \text{if } X \ge a$$

where: X = the value of x-axis (tree dbhob), a = location parameter, b = scale parameter, c = shape parameter.

The parameters of those equations can be calculated as shown below (Garcia, 1981)

$a =$ maximum dbhob (cm)

$c = \frac{1}{z\left(1+(1-z)^2 \sum_{i=0}^{5} k_i z^i\right)}$, and z and μ can be calculated as follows:

$z = \frac{\sigma}{a-\mu_a}$, $\mu_g = \sqrt{\frac{(4G)}{(\pi N)}}\,100$, $\mu_a = \sqrt{\mu_g{}^2 - \sigma^2}$

$b = \left[\frac{a-\mu_a}{\Gamma'(1+1/c)}\right]$, and $\Gamma'(1+x) = 1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$

Then, if we substitute $(1+x)$ with $(1+1/c)$, then $\Gamma'(1+1/c)$ can be written as follows:

$$\Gamma'(1+1/c) = 1 + a_1(1/c) + a_2(1/c)^2 + a_3 x(1/c)^3 + a_4(1/c)^4 + a_5(1/c)^5$$

where: $a_1 = -0.5748646$, $a_2 = 0.9512363$, $a_3 = -0.6998588$, $a_4 = 0.4245549$, $a_5 = -0.1010678$, $k_0 = -0.22004032$, $k_1 = -0.001433169$, $k_2 = 0.150611381$, $k_3 = -0.078575996$, $k_4 = -0.004305716$, $k_5 = -0.008804944$, σ = standard deviation of dbhob (cm), μ_g = mean quadratic dbhob (cm), μ_a = mean arithmetic dbhob (cm), G =stand basal area (m^2/ha), N = stand stocking (stems/ha) and a = maximum dbhob (cm).

Stand basal area and mortality functions have been developed in Chapter V. Hence, we only needed to generate the maximum dbhob (Dmax) and the standard deviation of dbhob (Dstd). The procedure used for previous stand-level models was applied to these variables by employing all-possible interval datasets. We used a proportion of 2:1 for the fitting and

validation dataset. The statistics that we used for comparing models were SEE, MRES, MAE, AIC for model fitting and SEE, MRES, and MAE for validation. The residual plots also became a major consideration. The smallest statistic was preferable for the chosen model. The models that tested for Dmax and Dstd are shown in Table 5 (Chapter III). The results of these models are shown in the following sections (6.3.1 and 6.3.2). R Software (R Core Team, 2013) was used for all data analysis process.

6.2.2. Validation of Reverse Weibull Distributions

We used the error index (Equation 6.1) proposed by Reynolds et al. (1988) for validating the diameter distribution model. However, since we only tested a single model of the reverse Weibull, we then used EI to compare the base model and augmented diameter distribution model (Casnati, 2016).

We used the original models of basal area, stocking, Dmax and Dstd without additional elevation and rainfall variables to generate the base diameter distribution model for all species. For the augmented diameter distribution model, we only used augmented basal area models for all species, augmented mortality, Dmax and Dstd models for *Acacia mangium* and an augmented Dmax model for *Eucalyptus pellita*. The use of augmented mortality, Dmax and Dstd models for *Acacia crassicarpa*, and mortality and Dstd models for *Eucalyptus pellita* are not recommended (see sections 5.5 and 6.3.2).

Error index (EI) can be calculated as follows (Reynolds et al., 1988):

$$EI = \sum_{i=1}^{k} w_i \left| f_i - \hat{f}_i \right| \qquad \text{(Equation 6. 1)}$$

where: i = diameter class (5 cm interval), w_i = the weight of class i, f_i = the actual stocking of class i, $\hat{f}_i$ = the predicted stocking of class i, EI =error index per plot

The predicted stocking of class i can be calculated as

$$\hat{f}_i = pd_i * \widehat{N}_t \qquad \text{(Equation 6. 2)}$$

where: pd_i = probability density of class i, and $\widehat{N}_t$ = predicted stand stocking at harvesting age (stems/ha).

We can use total volume (Reynold, 1988) or basal area (Peuhkurinen, Maltamo, & Malinen, 2008) as a weighting variable. Basal and total volume can be calculated by using mid-class

diameters of each class. Using a mid-class diameter alone as an input variable, we then need volume equations (Equations 3.4, 3.5 and 4.1) and height–dbhob equations to calculate total volume. However, all species in this study do not have a generalised height–dbhob equation; hence, we needed to construct this equation first (section 6.3.3).

6.3. Result

6.3.1 Standard Deviation of Diameter (Dstd) and Maximum Diameter (Dmax)

The best model for predicting the standard deviation of dbhob (Dstd) was Equation 9 (Table 4.3) for all species. Although the bias was not the smallest, Equation 9 gave a satisfactory result in other statistical values in terms of the smallest SE and MAE, both in model construction and validation for *Acacia mangium*. Equation 9 also gave superior results compared with others, and the bias of this equation was the smallest for *Eucalyptus pellita*. Meanwhile, Equation 9 had a contradictory result between the model construction and validation for *Acacia crassicarpa*. This equation performed the best in model construction but also performed the worst in the validation process. However, we chose Equation 9 because differences between it and other equations were small, and the residual plots of this equation (Figure 56) during model fitting looked similar to the others (Figure 55). Moreover, the histogram of this equation had a normal residual (*p* value of Shapiro–Wilk test = 0.1764). The statistical values of model fitting and validation for Dstd are shown in Tables 43 and 44.

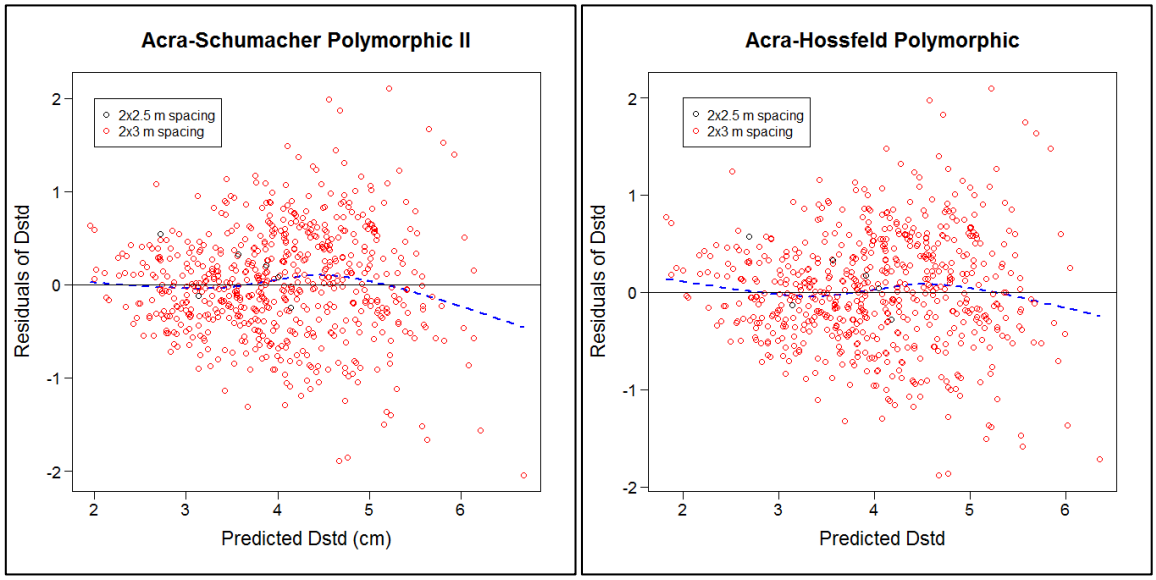

Figure 55 Scatterplots with lowess smoothing (dashed line) of residuals of standard deviation of diameter (Dstd) in model construction of Equation 4 (left) and Equation 21 (right) for *Acacia crassicarpa*

The best three equations for predicting maximum dbhob (Dmax) were Equations 4, 9 and 10 for *Acacia crassicarpa* and *Eucalyptus pellita*, and Equations 4, 9, and 21 for *Acacia mangium*. The statistical results of model fitting and validation are shown in Tables 45 and 46. From those tables, it was clear that Equation 9 and Equation 4 were superior to the other two equations for *Acacia mangium* and *Eucalyptus pellita* respectively. Although the mean bias of those equations did not give the best result, the residual plots of each equation were slightly better than others. We chose Equation 9 for *Acacia crassicarpa* due to a better residual plot, although the statistical values of validation for Equation 4 gave the best result compared with others.

Table 43 *Statistical values of model fitting for three best equations of Dstd*

Equation	Author/Name	Statistical values of model fitting			
		MRES	SEE	MAE	AIC
	Acacia mangium				
4	Schumacher/ Polymorphic II	−0.011 (2)	0.489 (2)	0.36 (2)	1700 (2)
9	von Bertalanffy –Richards Polymorphic I	−0.022 (3)	0.459 (1)	0.338 (1)	1547 (1)
21	Hossfeld Polymorphic	−0.01 (1)	0.491 (3)	0.362 (3)	1712 (3)
	Acacia crassicarpa				
4	Schumacher/ Polymorphic II	0.018 (3)	0.577 (3)	0.45 (2)	1110 (3)
9	von Bertalanffy –Richards Polymorphic I	0.007 (1)	0.571 (1)	0.453 (3)	1096 (1)
21	Hossfeld Polymorphic	0.016 (2)	0.574 (2)	0.448 (1)	1103 (2)
	Eucalyptus pellita				
4	Schumacher/ Polymorphic II	0.01 (2)	0.499 (3)	0.381 (3)	1332 (3)
9	von Bertalanffy –Richards Polymorphic I	0.004 (1)	0.493 (1)	0.372 (1)	1307 (1)
21	Hossfeld Polymorphic	0.012 (3)	0.497 (2)	0.379 (2)	1324 (2)

Note: Ranking is shown in brackets. This ranking only compared the best three models.

Table 44 *Statistical values of validation for three best equations for Dstd*

Equation	Author/Name	Statistical value of validation		
		MRES	SEE	MAE
	Acacia mangium			
4	Schumacher/ Polymorphic II	−0.008 (2)	0.469 (2)	0.347 (2)
9	von Bertalanffy–Richards Polymorphic I	−0.025 (3)	0.453 (1)	0.337 (1)
21	Hossfeld Polymorphic	−0.005 (1)	0.479 (3)	0.353 (3)
	Acacia crassicarpa			
4	Schumacher/ Polymorphic II	−0.129 (1)	0.508 (1)	0.387 (1)
9	von Bertalanffy–Richards Polymorphic I	−0.147 (3)	0.528 (3)	0.4 (3)
21	Hossfeld Polymorphic	−0.136 (2)	0.510 (2)	0.391 (2)
	Eucalyptus pellita			
4	Schumacher/ Polymorphic II	0.091 (2)	0.5054 (1)	0.388 (3)
9	von Bertalanffy–Richards Polymorphic I	0.079 (1)	0.5056 (2)	0.386 (1)
21	Hossfeld Polymorphic	0.096 (3)	0.5062 (3)	0.388 (2)

Note: Ranking is shown in brackets. This ranking only compared the best three models.

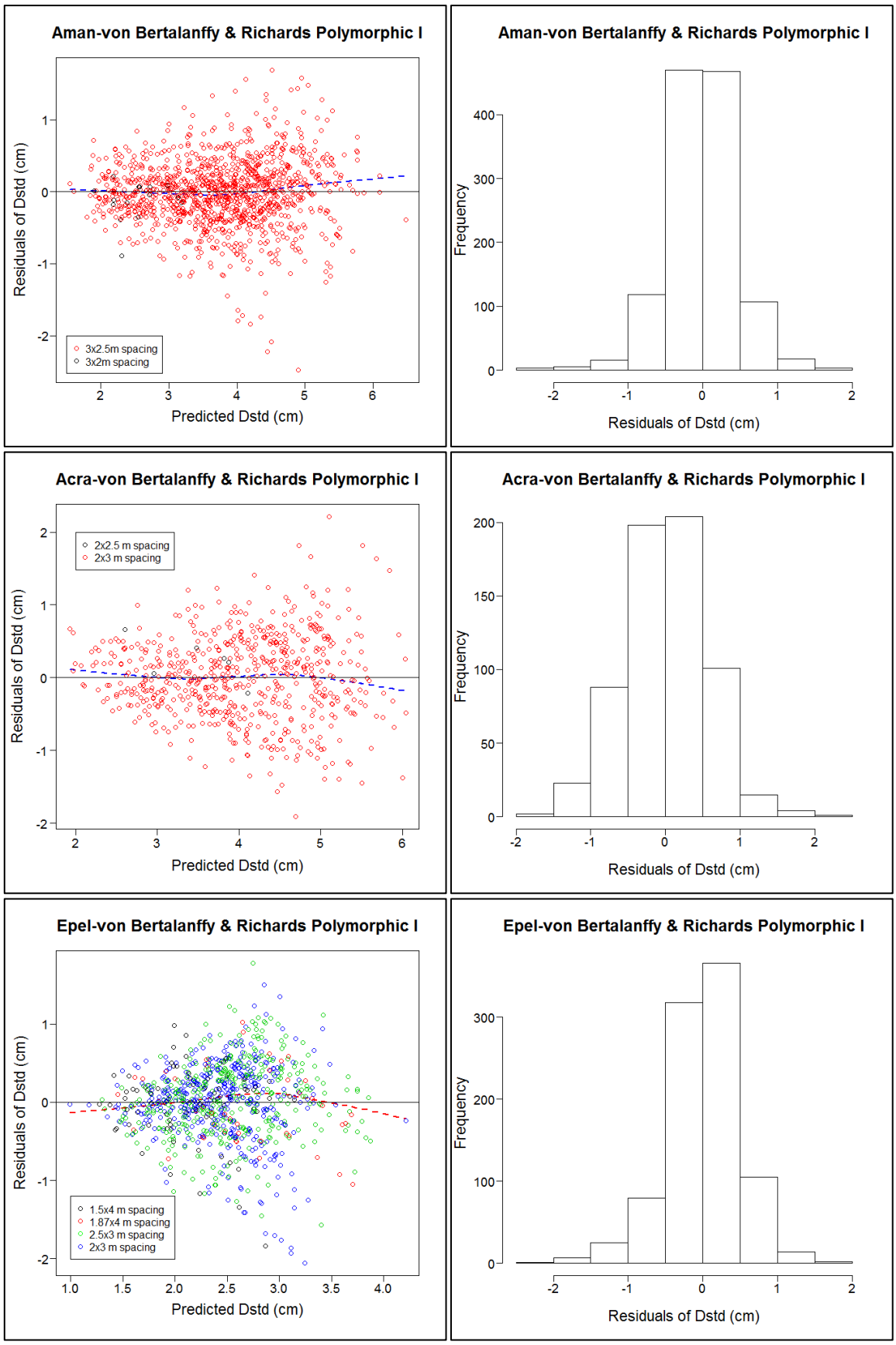

Figure 56 Scatterplots (left) with lowess smoothing (dashed line) and histogram (right) of residuals of standard deviation of diameter (Dstd) in model construction of Equation 9 for Dstd for *Acacia mangium* (top) and *Acacia crassicarpa* (middle) and for *Eucalyptus pellita* (bottom).

Table 45 *Statistical values of model fitting for three best equations of Dmax*

Equation	Author/Name	Statistical values of model fitting			
		MRES	SEE	MAE	AIC
	Acacia mangium				
4	Schumacher/ Polymorphic II	−0.05 (2)	2.241 (2)	1.657 (2)	5377 (2)
9	von Bertalanffy–Richards Polymorphic I	−0.051 (3)	2.129 (1)	1.59 (1)	5254 (1)
21	Hossfeld Polymorphic	−0.045 (1)	2.289 (3)	1.693 (3)	5428 (3)
	Acacia crassicarpa				
4	Schumacher/ Polymorphic II	0.042 (3)	2.132 (3)	1.521 (2)	2772 (3)
9	von Bertalanffy–Richards Polymorphic I	−0.011 (2)	2.011 (1)	1.48 (1)	2698 (1)
10	von Bertalanffy –Richards Polymorphic II	0.002 (1)	2.124 (2)	1.57 (3)	2767 (2)
	Eucalyptus pellita				
4	Schumacher/ Polymorphic II	0.03 (3)	1.283 (1)	0.924 (1)	3063 (1)
9	von Bertalanffy–Richards Polymorphic I	0.024 (2)	1.305 (2)	0.925 (2)	3094 (2)
10	von Bertalanffy–Richards Polymorphic II	0.021 (1)	1.305 (2)	0.95 (3)	3095 (3)

Note: Ranking is shown in brackets. This ranking only compared the best three models.

Table 46 *Statistical values of validation for three best equations for Dmax*

Equation	Author/Name	Statistical value of validation		
		MRES	SEE	MAE
	Acacia mangium			
4	Schumacher/ Polymorphic II	−0.334 (1)	1.857 (2)	7.112 (2)
9	von Bertalanffy –Richards Polymorphic I	−0.4 (3)	1.780 (1)	6.864 (1)
21	Hossfeld Polymorphic	−0.335 (2)	1.910 (3)	7.352 (3)
	Acacia crassicarpa			
4	Schumacher/ Polymorphic II	−0.222 (1)	1.996 (1)	1.569 (1)
9	von Bertalanffy –Richards Polymorphic I	−0.225 (2)	2.080 (3)	1.649 (3)
10	von Bertalanffy –Richards Polymorphic II	−0.338 (3)	2.020 (2)	1.591 (2)
	Eucalyptus pellita			
4	Schumacher/ Polymorphic II	0.256 (2)	1.361 (1)	1.006 (1)
9	von Bertalanffy –Richards Polymorphic I	0.241 (1)	1.374 (2)	1.021 (2)
10	von Bertalanffy –Richards Polymorphic II	0.285 (3)	1.392 (3)	1.022 (3)

Note: Ranking is shown in brackets. This ranking only compared the best three models.

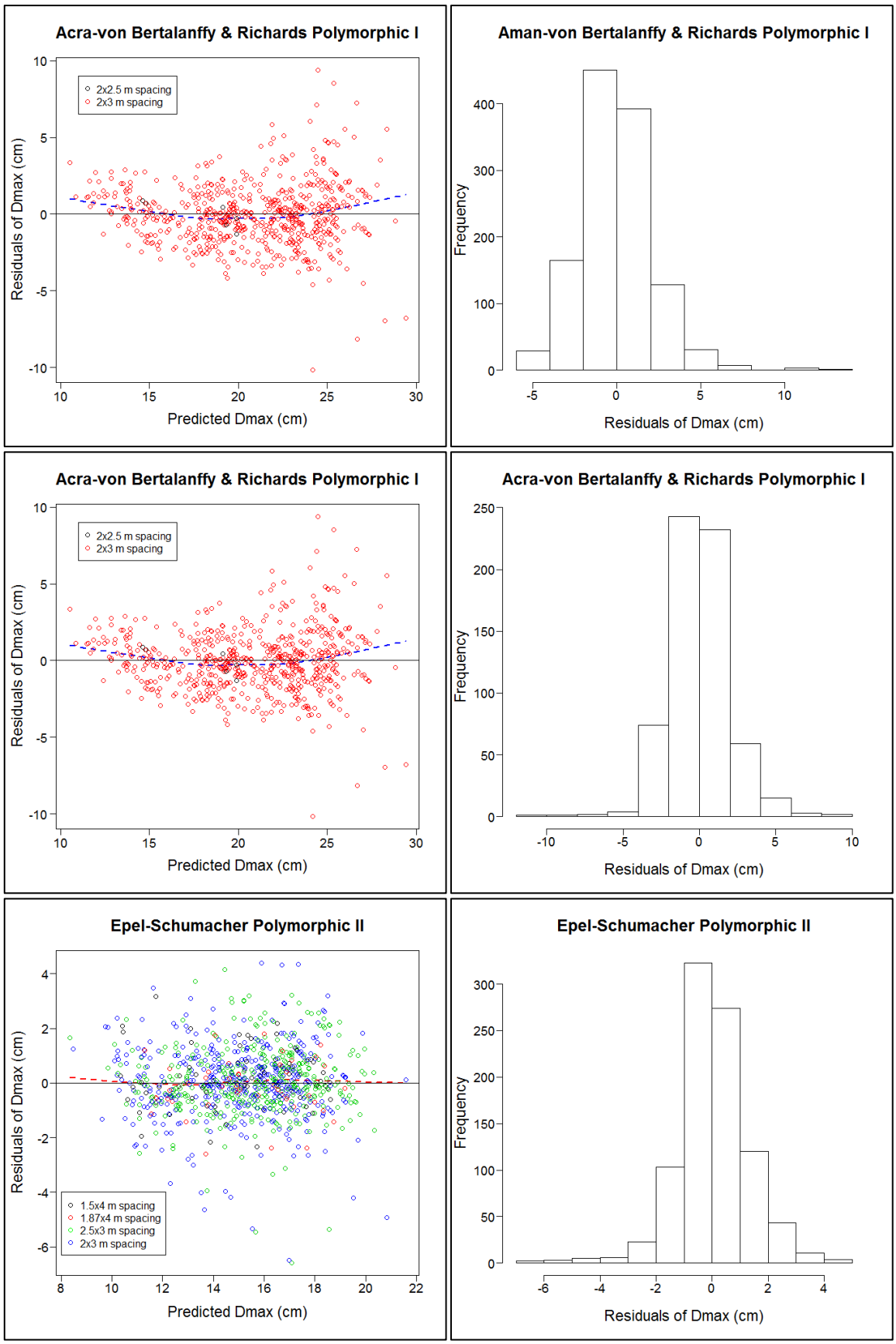

Figure 57 Scatterplots (left) with lowess smoothing (dashed line) and histogram (right) of residuals of maximum diameter (Dmax) in model construction of Equation 9 for *Acacia mangium* (top) and *Acacia crassicarpa* (middle), and Equation 4 for *Eucalyptus pellita* (bottom).

6.3.2 Augmented Models for Dmax and Dstd

Augmented models for Dmax and Dstd of *Acacia crassicarpa* only passed the parameter-checking process at α = 0.01and the reduction of SEE was very small (less than 1.15%), as we can see in Table 47. Dmax of *Eucalyptus pellita* passed the parameter-checking process, but on the other hand, Dstd equations of *Eucalyptus pellita* failed at that phase. Moreover, the reduction in SEE of Dmax was less than 0.1% for this species. Based on this consideration, we recommended that the company used the original model instead of the augmented model for both species.

Augmented models of Dmax and Dstd for *Acacia mangium* gave significant coefficients for augmentation variables (Appendix F) during the model-fitting and parameter-checking processes. The additional rainfall variable on parameters *b* and *c* gave a reduction of about 2.9% from the original model of Dmax and the additional variables of elevation and rainfall gave a 1.6% reduction in SEE for Dstd.

Table 47 *Standard error of augmented equations (with additional elevation, rainfall or temperature)*

Variable	SEE			Note	
	Original Equation	Augmented equation	Reduction	Variable added	into parameters
			Acacia mangium		
Dmax	2.129	2.068	2.86 %	Rainfall	*b* and *c*
Dstd	0.4586	0.4514	1.57%	Elevation and Rainfall	*b*
			Acacia crassicarpa		
Dmax	2.011	1.988[##]	1.14%	Elevation	*b* and *c*
Dstd	0.5712	0.5703**[##]	0.16%	Rainfall	*b*
			Eucalyptus pellita		
Dmax	1.283	1.272	0.85%	Elevation	*b* and *c*
Dstd	0.4925	0.4822[#]	2.09%	Elevation	*b* and *c*

*Note:***=significant at α = 0.01 in model fitting, [#]=insignificant parameters on parameter checking process, [##]=significant at α = 0.01 on parameter checking process*

We used Dmax of 8.9 cm starting at age 1 year for simulating projections for *Acacia mangium* to examine the effects of additional mean annual rainfall and elevation for *Eucalyptus pellita*. Both of the simulations showed that there was a turning point. The turning point for *Acacia mangium* was when the mean annual rainfall was around 2485 mm/year and elevation was 62 masl for *Eucalyptus pellita*. Dmax increased as the mean annual rainfall increased from 2395 to 2485 mm/year, and after that Dmax decreased as mean annual rainfall increased. From 12 to 62 masl, Dmax increased as the elevation increased, but after that, it started to decline for *Eucalyptus pellita* (Figure 58).

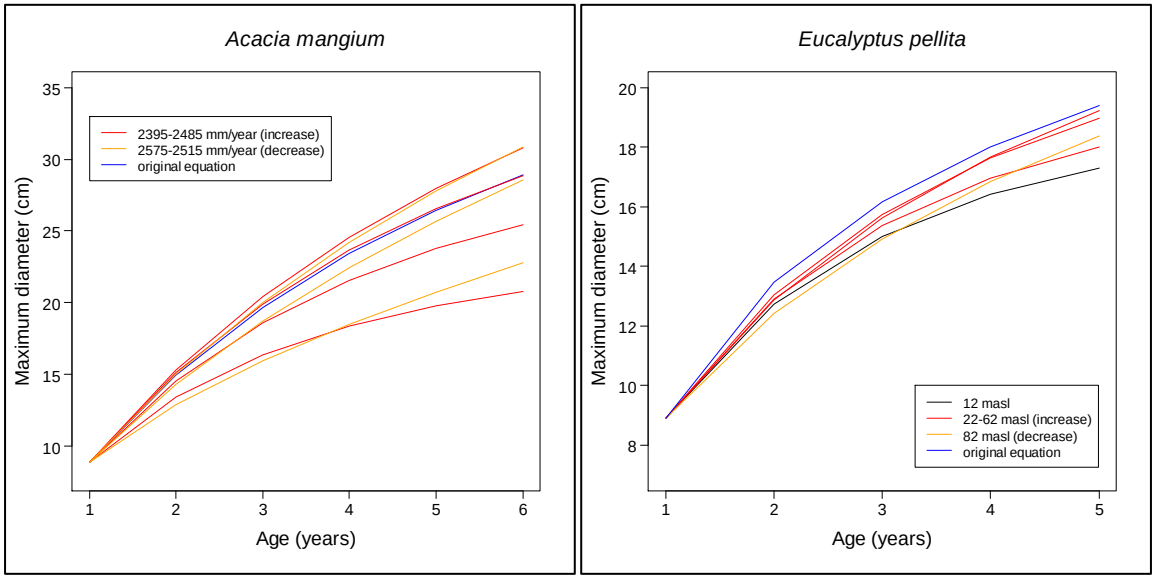

Figure 58 The projection of augmented equation of Dmax as an effect of mean annual rainfall (left) for *Acacia mangium* and as an effect of elevation (right) for *Eucalyptus pellita*

6.3.3 Generalised Height–dbhob Equations

We chose the two-parameter height–dbhob equations proposed by Näslund (1937) to be used in this study (section 4.2.3) due to its simplicity of in the field . We then used the parameter prediction approach using Equations 4.48 and 4.49 to construct generalised height–dbhob models for all species with additional stand variables that we obtained in Chapter V.

The results of the first step of the parameter prediction method are shown in Table 48. We then added all significant stand variables into parameters *a* and *b* of Equation 4.30. The reductions in SEE from this augmented Equation 4.30 are shown in Table 49 and the residual plots of model fitting and validation are shown in Figure 59.

The stand variables that affected Equation 4.30 are shown in Table 49, with *Eucalyptus pellita* having the fewest stand variables. On considering Table 49, it was clear that the augmented Equation 4.30 had a smaller SEE than the original one. The reductions in SEE were about 41%, 27% and 21% for *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita* respectively. Except for the bias for *Eucalyptus pellita*, other statistics of the generalised height–dbhob equation had smaller values than the base height–dbhob equation (Table 50). The parameters (Equations 4.48 and 4.49) of augmented Equation 4.30 are shown in Table 51. Although the residual plots of the generalised height–dbhob equations (Figure 59) did not seem better than the original (Figure 29), it seemed that most of the residuals points were moved closer to zero

than in the original. In conclusion, we recommended the companies to use the augmented Equation 4.30 (Table 51) as their generalised height–dbhob models.

Table 48 *The p values of the first step of parameter prediction method for all species*

Stand variables	*Acacia mangium*		*Acacia crassicarpa*		*Eucalyptus pellita*	
	a	***b***	***a***	***b***	***a***	***b***
Elevation (m)	0.03694	$< 2x10^{-16}$	0.964397*	0.00628	0.03054	0.00209
Basal Area (m^2/ha)	$3.47x10^{-07}$	0.00870	0.150163*	0.01522	0.08779**	0.15163*
Site Index (m)	0.54360*	0.03749	$1.92x10^{-06}$	$9.45x10^{-06}$	0.08305**	$5.06x10^{-05}$
Stocking (stems/ha)	0.65371*	$3.77x10^{-07}$	0.001626	$5.52x10^{-06}$	0.00152	$3.13x10^{-07}$
Age (years)	0.06444**	$4.52x10^{-11}$	0.906855*	0.13626*	0.70811*	0.38430
Stocking:Age	0.01192	$7.67x10^{-08}$	0.026191	0.00279	0.20406	0.00159

*Note : *= insignificant parameters, **=significant at α=0.01*

Table 49 *Standard error of augmented height–dbhob model (Equation 4.30) for all species*

Species	SEE			Stand variables added	
	Original equation	Augmented equation	Reduction	a	b
Acacia mangium	1.986	1.181	40.53%	E, G, NA	E, G, SI, N, A, NA
Acacia crassicarpa	2.482	1.823	26.55%	SI, N, NA	E, G, SI, N, NA
Eucalyptus pellita	2.32	1.815	21.77%	E, N	E, SI, N, NA

Note: E = elevation (m), G=basal area (m^2/ha), SI= site index (m), N =stocking (stems/ha), A = age (years), NA=stocking x age.

Table 50 *The comparison of statistical values of validation between the base height–dbhob models and the generalised height–dbhob models*

Species	Equation	Model fitting				Validation		
		MRES	SEE	MAE	AIC	MRES	RMSE	MAE
Acacia mangium	Original	−0.006	1.986	1.502	139890	−0.036	1.999	1.513
	Augmented	−0.005	1.181	0.890	107240	−0.025	1.186	0.898
Acacia crassicarpa	Original	0.040	2.482	1.876	56749	0.089	2.506	1.898
	Augmented	−0.001	1.823	1.329	49243	0.006	1.806	1.334
Eucalyptus pellita	Original	0.028	2.320	1.821	78620	0.026	2.348	1.837
	Augmented	−0.073	1.815	1.409	70076	−0.079	1.822	1.414

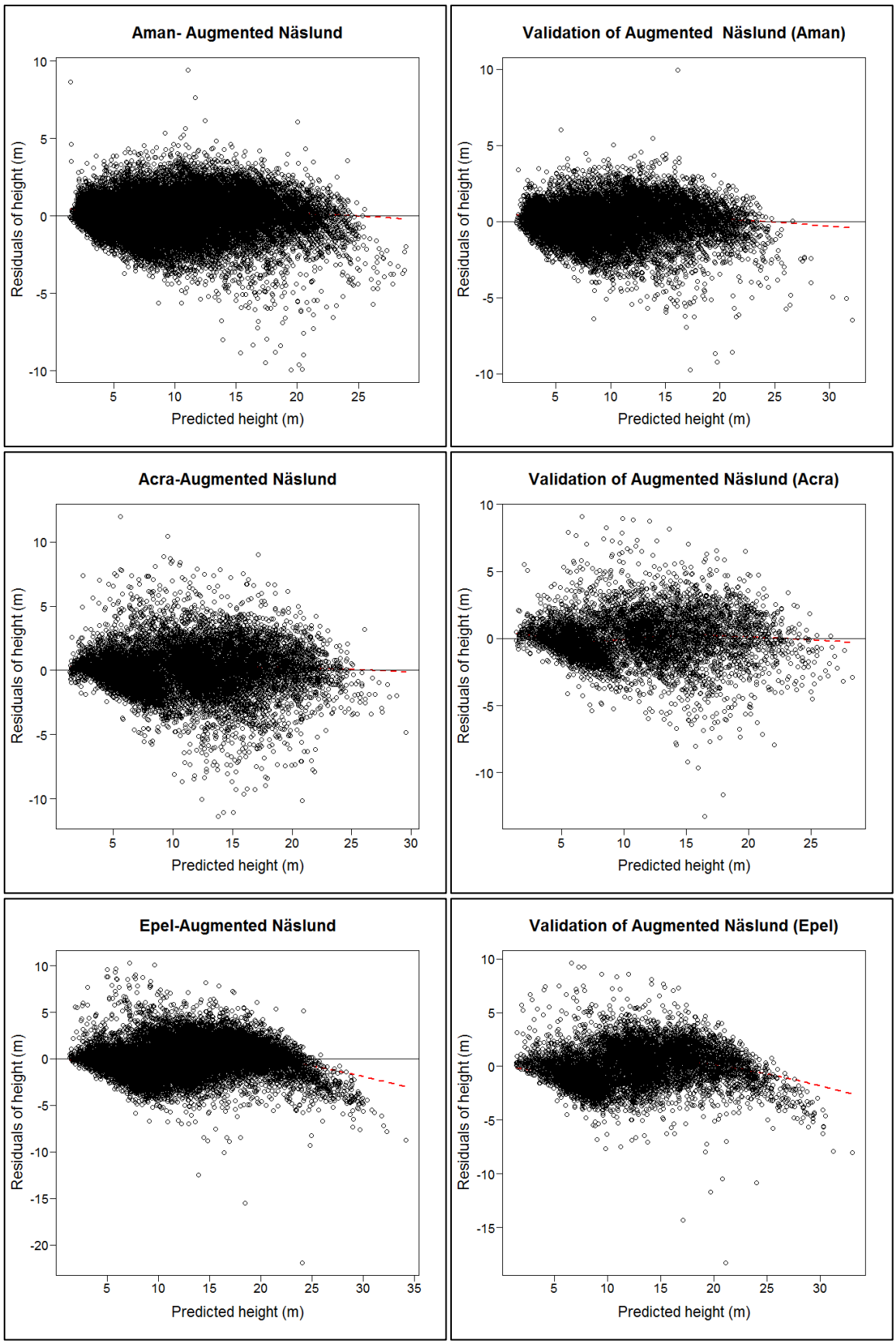

Figure 59 Residual plots of generalised height–dbhob of Equation 4.30 in model fitting (left) and validation (right) for *Acacia mangium* (top), *Acacia crassicarpa* (middle) and *Eucalyptus pellita* (bottom).

Table 51 *Coefficients of the augmented height–dbhob models for all species*

Parameters	*Acacia mangium*	*Acacia crassicarpa*	*Eucalyptus pellita*	Note
a_0	2.17	2.639	1.321	
a_1	−0.001502		−0.001923	Elevation
a_2	−0.04187			Basal area
a_3		−0.04672		Site Index
a_4		−0.00004112	−0.0001005	Stocking
a_5				Age
a_6	−0.00004812	−0.000151		Stocking × Age
b_0	0.2678	0.2258	0.2722	
b_1	0.00008032	−0.0002305	0.0001864	Elevation
b_2	0.002897	0.0002601		Basal area
b_3	−0.00555	−0.003026	0.00003143	Site Index
b_4	0.00004943	0.00004454	0.00003143	Stocking
b_5	−0.005647			Age
b_6	−0.00001041	−0.000003457	−0.00001018	Stocking × Age

6.3.4 Validation of Diameter Distribution Models

We used the first measurement from the validation dataset with 2-cm intervals of diameter classes as a starting value to project the diameter distributions at age 5 years. The starting variables were stocking (N_1), age (T_1), basal area (G_1), maximum diameter ($Dmax_1$), standard deviation of diameter ($Dstd_1$) at first measurement for all species. We also needed mean top height (MTH_1) at first measurement to calculate the site index at age 5 years (SI). Particularly for *Acacia mangium*, SI was used to calculate the probability density function of mortality for *Acacia mangium* (Table 29). From those variables, we obtained stocking (N_5), basal area (G_5), maximum diameter ($Dmax_5$), standard deviation ($Dstd_5$), mean quadratic diameter (μ_{q5}) and mean arithmetic diameter (μ_{a5}) at age 5 years. After that, the parameters of the reverse Weibull distribution were obtained in order to calculate the cumulative density (cd) of each diameter class (i) in order to get the probability density (pd) at a particular diameter class (i). The probability density of each class (pd_i) was calculated as follows (Equation 6.3):

$$pd_i = cd_i - cd_{i-1} \quad \text{(Equation 6. 3)}$$

The cumulative density of each class (cd_i) was calculated as a subtraction from the cumulative density at the upper-class diameter of class i by the cumulative density at upper-class diameter

of previous class ($i-1$). The cumulative density of each class (cd_i) is written below (Equation 6.4):

$$cd_i = \sum_{i-1}^{i}\left[exp\left(-\left(\frac{a-upD_i}{b}\right)^c\right)\right] \quad \text{(Equation 6. 4)}$$

where: a, b, c = parameters of reverse Weibull distribution, i = diameter class,

upD_i = upper diameter at i class, upD_{i-1} = upper diameter at previous $(i-1)$ class.

Table 52 Error indices of the base and augmented models of diameter distribution

Statistics	Base model			Augmented model			Note
	Stems/ha	m^2/ha	m^3/ha	Stems/ha	m^2/ha	m^3/ha	
Acacia mangium							
Min	196	3.60	29.17	201	4.00	33.20	45 plots
Mean	372	8.34	71.91	376	8.39	73.33	
Max	759	19.83	169.53	825	20.28	179.20	
Acacia crassicarpa							
Min	212	5.30	49.28	194	5.00	48.51	19 plots
Mean	448	9.41	85.84	448	9.44	86.10	
Max	725	17.48	164.75	758	17.42	166.85	
Eucalyptus pellita							
Min	294	4.60	41.68	277	4.67	49.38	31 plots
Mean	659	9.43	103.03	665	9.46	102.70	
Max	2,187	16.07	182.39	2,196	15.97	166.74	

Casnati (2016) calculated the error index based on stocking values (without weighting) and she found that the mean error indices were 347 and 327 stems/ha for *Pinus taeda* and *Eucalyptus grandis* respectively. Mehtätalo et al. (2015) found that the error indices ranged from 450 to 605 for diameter distribution for boreal forest in Finland. Our error indices were 372, 448 and 659 stems/ha for the base models of *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita* respectively. In term of stocking/ha, the mean absolute difference between the original and augmented models for *Acacia crassicarpa* had a similar result (Table 52). Augmented models were only slightly smaller (0.6%) and slightly larger (1.08%) than the base models for *Eucalyptus pellita* and *Acacia mangium* respectively. Error indices of the augmented models were slightly larger than the original one for both *Acacia* species, and in contrast, error indices of the augmented models were slightly smaller than the original model for *Eucalyptus pellita*. The differences of mean error indices were less than 2 m^2/ha for all species (Table 52).

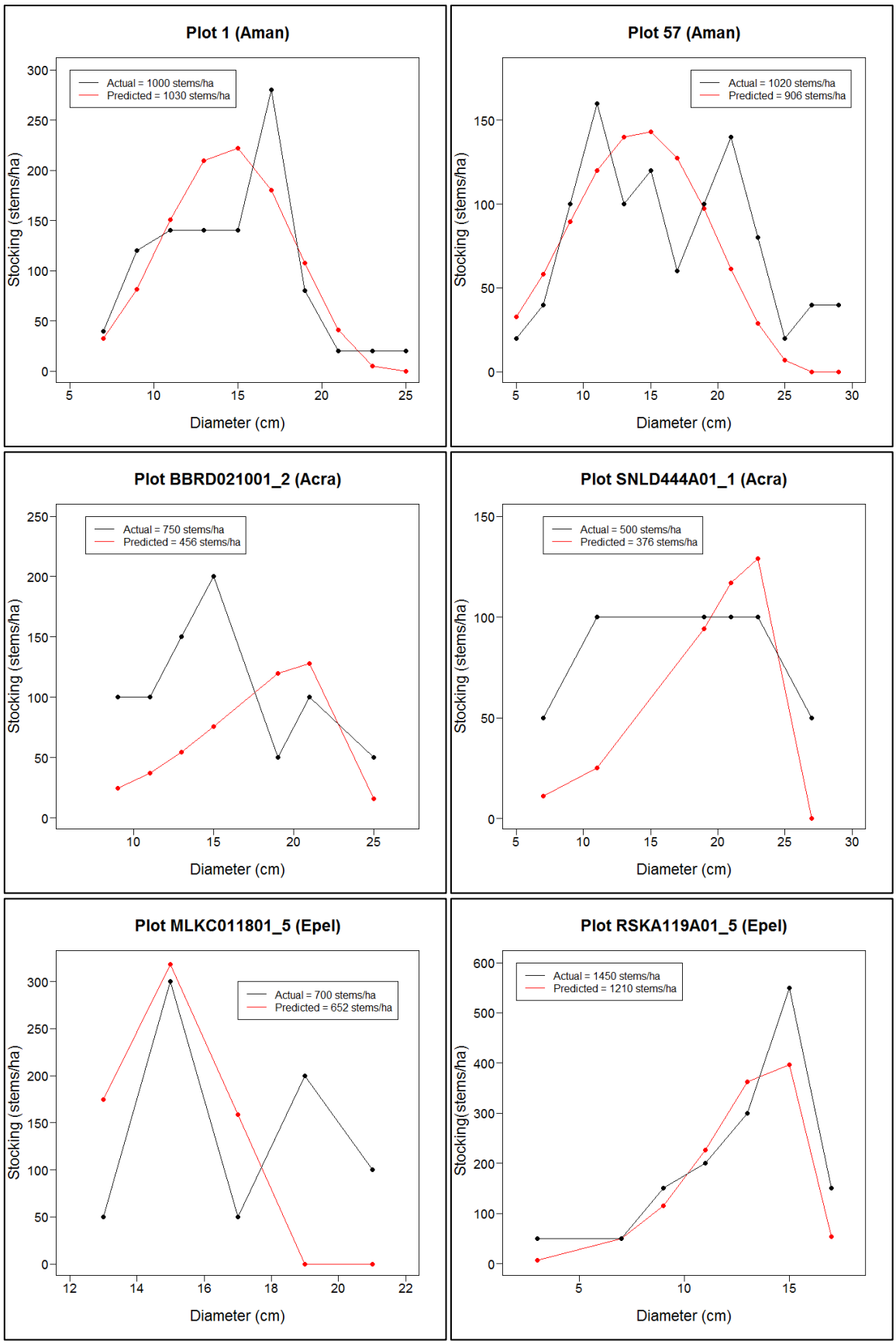

Figure 60 Some plots that were randomly selected from the validation dataset for diameter distribution projection

Some researchers (Bailey & Dell, 1973; Frazier, 1981; Mason & Whyte, 1997; Methol, 2001) used frequency plots to see the accuracy of diameter distribution models, instead of using the error index. We randomly selected two plots of each species to see the difference between the projection and the actual diameter distribution at age 5 years (Figure 60). From those figures, we can see that *Acacia crassicarpa* had the poorest projection compared with other species. The stocking projection for *Eucalyptus pellita* and *Acacia mangium* had small differences compared with the actual distributions.

6.4. Discussion and conclusions

A diameter distribution model was used to estimate the number of trees for dbh classes of a stand. Furthermore, by using volume equations and the mid-point of each diameter class, the total volume of each dbh class can be projected. For commercial purposes, this estimation will help forest managers to get information about the commercial log from larger trees. This, of course, gives a better visualisation than total stand-volume projection.

We generated maximum dbhob (Dmax) and standard deviation of dbhob (Dstd) to generate diameter distributions models for *Acacia mangium, Acacia crassicarpa* and *Eucalyptus pellita*. We found that the von Bertalanffy–Richards Polymorphic (Equation 9) was the best equation for Dstd prediction for all species. The standard error of Dstd ranged from 0.45 to 0.53 for all species. The best models of Dmax were the von Bertalanffy–Richards Polymorphic (Equation 9) for both *Acacia* species and the two-parameter Schumacher Polymorphic for *Eucalyptus pellita*. The standard errors of Dmax were around 2 cm for both *Acacia* and around 1.3 cm for *Eucalyptus pellita*. Von Bertalanffy–Richards Polymorphic was also the best model for projecting Dstd, and the two-parameter Schumacher Polymorphic was also the best model for Dmax projection for *Pinus radiata* in New Zealand (Methol, 2001), *Pinus taeda* and *Eucalyptus grandis* in Uruguay (Casnati, 2016).

Augmented models of Dmax and Dstd have slightly improved on the original models. For *Acacia mangium*, the reductions in SEE were greater than 1.5% by adding a rainfall variable into the original Dmax, and adding both elevation and rainfall into the original Dstd equation. Augmented Dmax and Dstd models of *Acacia crassicarpa*, and an augmented Dstd model of *Eucalyptus pellita* did not pass the parameter-checking process. Meanwhile, an augmented Dmax model for *Eucalyptus pellita* had less than 1% reduction in SEE compared with the

original. Hence, we recommended that augmented Dmax and Dstd models not be used for *Acacia crassicarpa* and *Eucalyptus pellita*.

Validation of diameter distribution models were achieved using an error index proposed by Reynolds et al. (1988) that employs basal area and volume as weighting factors. Some plots (Bailey & Dell, 1973; Frazier, 1981; Mason & Whyte, 1997; Methol, 2001) were also randomly selected to see the accuracy between the actual and projection at age 5 years (Figure 60). The mean error indices were noted around 72, 86 and 103 m^3/ha for *Acacia mangium*, *Acacia crassicarpa* and *Eucalyptus pellita* respectively for the original diameter distribution models. The mean error indices of augmented diameter distribution models were only slightly different (less than 2 m^3/ha) from the original model for all species. Hence, the use of augmented models needs to be reviewed thoroughly by considering their simplicity.

From the frequency plots, the projection of *Acacia crasssicarpa* performed the worst, with a difference between actual and predicted stocking up to 40%. However, the mean absolute difference between actual and predicted stocking was around 448 stems/ha for this species. On the other hand, *Eucalyptus pellita* had the largest mean absolute difference of stocking (659 stems/ha) among the other two species. When we studied the frequency plots in Figure 60, it seemed like the big differences in stocking occurred in the upper diameter class for *Eucalyptus pellita*. With the application of a weighting factor, it was clear why this species had the largest error indices. What is notable is that the difference between projection and actual distribution was influenced by the mortality equations. If the projections of distribution are very different from the actual values, the failure lies in mortality projection, not in the diameter distribution, since the diameter distribution model was estimating the probability function of each diameter class based on mortality models.

The best two-parameter model of the height–dbhob relationship was the Näslund equation with the power value equal to −2 (Equation 4.30) for all species. In this study, we also found that the best three-parameter height–dbhob model equation for all species was the Gompertz equation (Equation 4.43). However, due to its simplicity of use and small differences in error between those equations, we choose the Näslund equation instead of the Gompertz equation. Krisnawati et al. (2010) tested six equations from the three-parameter height–dbhob models and found that the best equation for the height–dbhob model for *Acacia mangium* in South Sumatra, Indonesia was the Lundqvist–Korf with 1.9 metres of RMSE. The generalised height–diameter equation was created by adding stand variables such as elevation, basal area/ha, site

index, stocking/ha, and age. Using a parameter prediction method, the stand variables that affected height varied across the species. The standard error of the generalised height–diameter model for *Acacia mangium* was 1.2 metres and around 1.8 metres for both *Acacia crassicarpa* and *Eucalyptus pellita*.

CHAPTER VII

Summary of Conclusions

7.1. General Conclusion

7.1.1. Taper and Volume for *Acacia mangium*

A general combined variable with scaled power transformations (Equation 4.11) was selected for predicting the volume of *Acacia mangium*, and a fourth-degree polynomial equation was chosen as the best taper equation for this species. The scaled power transformation performed well in complying with the constant variance of errors assumption in regression and also gave smaller errors compared with the weighted least-squares method. The fourth-degree polynomial was chosen due to its simplicity compared with fifth-degree polynomial taper equations. The standard error of the selected volume equation was about 0.02 m^3 and 1 cm for the taper equation. The total volume equation, taper equation and merchantable volume are written below, and the parameters of this equation can be seen in Tables 15, 21 and 22.

a. Total volume (v)

$$v = \left(\lambda_v \left(\beta_0 + \beta_2 \frac{h_t^{\lambda_h} - 1}{\lambda_h} + \beta_3 \frac{\left(dbhob^2 h_t\right)^{\lambda_{d^2h}} - 1}{\lambda_{d^2h}} \right) + 1 \right)^{\frac{1}{\lambda_v}}$$

b. Taper equation

$$d^2 = \frac{v_t}{k\, h_t} \left(2\beta_1 z^1 + 3\beta_2 z^2 + 4\beta_3 z^3 + 5\beta_4 z^4 \right)$$

c. Merchantable volume

$$v_{mv} = v_t \int_h^0 \left[\beta_1 \left(\frac{h_t - h}{h_t} \right)^2 + \beta_2 \left(\frac{h_t - h}{h_t} \right)^3 + \beta_3 \left(\frac{h_t - h}{h_t} \right)^4 + \beta_4 \left(\frac{h_t - h}{h_t} \right)^5 \right]$$

7.1.2. Height and Diameter

The selected height–dbhob model was Näslund's (Equation 4.30) for all species. The two-parameter Näslund model was chosen due to its simplicity of application in the field compared with the three-parameter equations. Moreover, Näslund's had the smallest AIC, MAE and SEE among the other two-parameter models. The stand variables that affected generalised height–dbhob equations varied among species. The standard error of generalised height–dbhob model

for *Acacia mangium* was 1.2 metres and around 1.8 metres for both *Acacia crassicarpa* and *Eucalyptus pellita*. The parameters of this equation can be seen in Table 51.

$$h_t = bh + \frac{dbhob^2}{(a + b\,dbhob)^2}$$

7.1.3. Growth and Yield Modelling

7.1.3.1. Empirical Models

The choice of selected models was based on a set of criteria, such as having a smaller error (SEE, RMSE, MAE), lack of bias (MRES) and having a distribution of residuals that were close to normal. In this study, we only chose the model that passed the parameter checking in order to minimise the pseudoreplication effect. Moreover, we selected the model after we ran simulations to see if the models met the biological realism and we also considered the simplicity of a model to be applied in the field. The selected models are described below.

A von Bertalanffy–Richards Polymorphic (Equation 9) was the best equation for mean top height (MTH) projections for all species. The range of standard errors for this equation was around 1.3 m for both species of *Acacia* and 2 m for *Eucalyptus pellita*. The parameters of this equation can be seen in Tables C1, D1 and E1 (Appendices C–E).

$$H_2 = b\left(\frac{H_1}{b}\right)^{\frac{\ln(1-e^{-cT_2})}{\ln(1-e^{-cT_1})}}$$

A two-parameter Schumacher Polymorphic (Equation 4) was the best equation for the basal area projection for all species. The standard errors were 2.2, 2.3 and 2.5 m^2/ha for *Acacia mangium, Acacia crassicarpa* and *Eucalyptus pellita* respectively. The parameters of this equation can be seen in Tables C2, D2, and E2 (Appendices C–E).

$$G_2 = e^{\ln(G_1)\left(\frac{T_1}{T_2}\right)^c + b\left(1-\left(\frac{T_1}{T_2}\right)^c\right)}$$

A two-step regression procedure (Woollons, 1998) was selected for projecting mortality because it produced the smallest bias compared with other approaches. Mortality equations were specific for each species and the standard errors were 114, 139 and 134 stems/ha for *Acacia mangium, Acacia crassicarpa* and *Eucalyptus pellita* respectively. The parameters of this equation can be seen in Tables C5, D5, and E5 (Appendices C–E) and the parameters of the probability function can be seen in Table 29. The mortality equations used were:

a. Weibull anamorphic (Equation 11) for *Acacia crassicarpa*

$$N_2 = N_1 \left(\frac{1 - e^{-bT_2^c}}{1 - e^{-bT_1^c}} \right)$$

b. Exponential decay anamorphic (Equation 30) for *Acacia mangium*

$$N_2 = N_1 e^{-b(T_2 - T_1)}$$

c. Two-parameter Schumacher Polymorphic (Equation 4) for *Eucalyptus pellita*

$$N_2 = e^{\ln(N_1)\left(\frac{T_1}{T_2}\right)^c + b\left(1 - \left(\frac{T_1}{T_2}\right)^c\right)}$$

The best model for estimating standard deviation of dbhob (Dstd) was the von Bertalanffy–Richards Polymorphic (Equation 9) for all species. The standard error of Dstd ranged from 0.45 to 0.53 for all species. The parameter of this equation can be seen in Tables C6, D6, and E6 (Appendices C–E):

$$Dstd_2 = b \left(\frac{Dstd_1}{b} \right)^{\frac{\ln(1 - e^{-cT_2})}{\ln(1 - e^{-cT_1})}}$$

The best models for estimating maximum dbhob (Dmax) were the von Bertalanffy–Richards Polymorphic (Equation 9) for *Acacia mangium* and *Acacia crassicarpa*, and the two-parameter Schumacher Polymorphic (Equation 4) for *Eucalyptus pellita*. The standard errors of Dmax were around 2 cm for both species of *Acacia* and around 1.3 cm for *Eucalyptus pellita*. The parameters of this equation can be seen in Tables C7, D7, and E7 (Appendices C–E). The Dmax equations used were:

a. von Bertalanffy–Richards Polymorphic (Equation 9) for both *Acacia* species

$$Dmax_2 = b \left(\frac{Dmax_1}{b} \right)^{\frac{\ln(1 - e^{-cT_2})}{\ln(1 - e^{-cT_1})}}$$

b. two-parameter Schumacher Polymophic (Equation 4) for *Eucalyptus pellita*

$$Dmax_2 = e^{\ln(Dmax_1)\left(\frac{T_1}{T_2}\right)^c + b\left(1 - \left(\frac{T_1}{T_2}\right)^c\right)}$$

The best stand-volume equation was Equation 2.8, which employs basal area/ha (*G*) and basal area × mean top height (*GH*) as input variables:

$$V = G(\alpha_0 + \alpha_1 H)$$

The standard errors of this equation were 2.1, 7.2 and 3.3 m^3/ha for *Acacia mangium, Acacia crassicarpa* and *Eucalyptus pellita* respectively. The parameters of this equation can be seen in Table 37.

7.1.3.2. Augmented Models

Effect of elevation on MTH gave reductions in standard error of about 1.4% for *Acacia mangium*, 0.4% for *Acacia crassicarpa* and 1.1% for *Eucalyptus pellita* compared with the empirical models. From the simulation process, both *Acacia mangium* and *Eucalyptus pellita* had a turning point: MTH at lower elevation increased as the elevation increased, and after it reached the turning point (40 masl for *Eucalyptus pellita* and 90 masl for *Acacia mangium*), it gradually decreased as the elevation increased. In contrast, the simulation of MTH showed a very small effect for *Acacia crassicarpa*.

Mean annual rainfall was found to have an effect on basal area for *Acacia crassicarpa* and *Eucalyptus pellita*, and basal area was affected by elevation for *Acacia mangium.* The reductions of augmented models by adding those variables were around 3.3%, 4.5% and 1.1% for *Acacia mangium, Acacia crassicarpa* and *Eucalyptus pellita* respectively. Basal area increased as elevation increased for *Acacia mangium*, and it decreased as the mean annual rainfall increased for *Acacia crassicarpa*. The effect of mean annual rainfall on basal area had a turning point around 2450 mm/year for *Eucalyptus pellita*.

The augmented mortality models were not recommended due to insignificant parameters during the parameter-checking process for *Acacia crassicarpa* and *Eucalyptus pellita*. Standard error for *Acacia mangium* was reduced by 2.3% for mortality by adding an elevation variable. Stocking increased with elevation from 10-184 masl.

Augmented models of Dmax and Dstd gave reductions in standard error of about 2.9% by adding mean annual rainfall, and 1.6% by adding elevation and mean annual rainfall variables for *Acacia mangium*. Augmented Dmax and Dstd models for *Acacia crassicarpa* and *Eucalyptus pellita* were not recommended due to their producing the smallest reduction

compared with the empirical models, and insignificant parameters during the parameter-checking process. Parameters of augmented models are shown in Table F.4 (Appendix F)

7.2. Limitations and Applicability

There were some limitations to this study. Firstly, the models that were created for *Acacia mangium* ignored the singling treatment (cutting competitor stems at early growth). This treatment certainly affected the mortality models, but this treatment was not well recorded. Secondly, the individual tree volume that was used for *Acacia crassicarpa* was from the equation for *Acacia mangium* that had previously been planted in that area. The company is still using this equation and a stems analysis has not been carried out to generate the volume equation for *Acacia crassicarpa*.

The stand level and diameter distribution models that were created from this study are better being used within the range of variables that are shown in Table 3. The use of augmented models had to be carefully applied. An additional input of elevation or a climatic variable should address the feasibility of forest inventory practice in the field.

7.3.Recommendation for Further Research

Based on what we observed during this study, we identified some potential research that may be useful for growth and yield study in the study area. Those recommendations are to:

1. carry out a stem analysis for *Acacia crassicarpa* to generate compatible taper and volume equations. In addition, based on the volume dataset for *Eucalyptus pellita*, a compatible taper equation can be developed.
2. create a bark model particularly for *Acacia mangium*, to improve the existing merchantable volume for under bark.
3. develop experimental plots to inspect the effect of site variables or silvicultural treatments on tree growth.
4. develop hybrid models as a bridge from a stand-level models and Physiological Processes Predicting Growth (3PG) model (Landsberg & Waring, 1997) for those species.

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APPENDICES

Appendix A. Statistical value of volume under bark equations

Table A. 1 Statistical values of model fitting and validation of volume under bark

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.9	0.026 (5)	6.57 x 10^{-03} (6)	0.01852 (6)	0.03467754 (6)	0.00876 (6)	0.20176 (6)
4.10	0.02423 (4)	-5.92 x 10^{-18} (2)	0.01708 (4)	0.03261333 (4)	0.0022 (1)	0.19768 (2)
4.11#	0.02093 (3)	-4.02 x 10^{-18} (1)	0.01449 (3)	0.02880789 (3)	0.0022 (1)	0.1979 (3)
4.12	0.01866 (2)	1.04 x 10^{-03} (4)	0.01257 (2)	0.02754487 (1)	0.00324 (4)	0.19858 (4)
4.13	0.01848 (1)	8.87 x 10^{-10} (3)	0.01244 (1)	0.02773933 (2)	0.0022 (3)	0.19751 (1)
4.14*	0.02606 (6)	5.92 x 10^{-03} (5)	0.01814 (5)	0.03466988 (5)	0.00811 (5)	0.2014 (5)

*Note: #=after removing D^2, *= some parameters in models were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table A. 2 Statistic value of weighted least-squares method of volume under bark

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.9	0.02953 (3)	-3.24 x 10^{-17} (2)	0.01884 (6)	3.825424 (6)	0.0022 (1)	0.20498 (6)
4.10	0.02677 (2)	-5.57 x 10^{-17} (3)	0.017 (4)	3.561715 (4)	0.0022 (1)	0.20203 (4)
4.11#	0.02308 (1)	2.14 x 10^{-17} (1)	0.01411 (3)	3.043755 (3)	0.0022 (1)	0.19837 (3)
4.12	0.09498 (5)	2.17 x 10^{-04} (6)	0.01246 (2)	2.745278 (1)	0.00241 (6)	0.19817 (2)
4.13	0.09438 (4)	1.23 x 10^{-04} (4)	0.01221 (1)	2.750365 (2)	0.00232 (4)	0.19797 (1)
4.14*	0.1302 (6)	1.83 x 10^{-04} (5)	0.01768 (5)	3.681309 (5)	0.00238 (5)	0.20236 (5)

*Note: #=after removing D^2, *= some parameters in models were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table A. 3 Statistic value of transformation of four best model of volume under bark

Equation	Model fitting**			Validation**		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.10	0.02258 (4)	5.12 x 10^{-04} (2)	0.01455 (4)	0.2607698 (4)	0.00662 (2)	0.20465 (4)
4.11#	0.01877 (2)	6.34 x 10^{-04} (3)	0.01225 (2)	0.2587335 (2)	0.0071 (3)	0.20375 (2)
4.12	0.02036 (3)	1.68 x 10^{-03} (4)	0.0131 (3)	0.02775652 (1)	0.00557 (1)	0.20317 (1)
4.13	0.01867 (1)	4.09 x 10^{-04} (1)	0.01208 (1)	0.2594894 (3)	0.00734 (4)	0.20407 (3)

*Note: #=after removing D^2, **= the value after back-transform,*
Ranking is shown in brackets. (The smallest rank was preferable).

Appendix B Taper equations

Tabel B. 1 Statistical values of model fitting and validation of incompatible taper functions (under bark).

Equation	Model fitting*			Validation*		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.15	1.029 (2)	0.141 (2)	0.8198 (2)	1.0393 (2)	0.0822 (1)	0.8 (1)
4.16	1.027 (1)	0.1912 (4)	0.8252 (3)	1.0372 (1)	0.1776 (3)	0.8303 (3)
4.17	1.0502 (3)	-0.1703 (3)	0.793 (1)	1.0668 (3)	-0.1828 (4)	0.8128 (2)
4.18	1.0669 (4)	0.2859 (5)	0.858 (4)	1.1072 (4)	0.2334 (5)	0.8852 (4)
4.19	1.1829 (5)	-0.0795 (1)	0.9335 (5)	1.2421 (5)	-0.1771 (2)	0.9064 (5)

*Note: * = the statistical value of d, therefore we get comparison equally among functions.*
Ranking is shown in brackets. (The smallest rank was preferable).

Tabel B. 2 Statistical values of model fitting and validation of compatible taper functions (under bark).

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.20*	25.11 (8)	-3.974 (3)	17.307 (8)	22.793 (8)	-5.368 (9)	16.305 (9)
4.21	24.39 (4)	-4.88 (6)	16.522 (1)	21.627 (7)	-5.076 (6)	15.427 (4)
4.22*	24.14 (1)	-3.576 (1)	16.57 (2)	21.544 (4)	-4.154 (2)	15.574 (8)
4.23*	24.15 (2)	-3.64 (2)	16.579 (3)	21.495 (3)	-4.194 (3)	15.492 (5)
4.24	24.55 (6)	-5.472 (8)	16.666 (7)	21.614 (5)	-5.167 (7)	15.505 (7)
4.25*	24.56 (7)	-5.483 (9)	16.636 (5)	21.623 (6)	-5.224 (8)	15.493 (6)
4.26	24.2 (3)	-4.219 (4)	16.59 (4)	21.356 (2)	-4.536 (5)	15.205 (3)
4.27	28.9 (10)	-5.435 (7)	20.738 (10)	26.566 (10)	-4.264 (4)	19.472 (11)
4.28	26.48 (9)	-7.016 (10)	18.203 (9)	23.564 (9)	-5.759 (10)	17.03 (10)
4.29	24.49 (5)	-4.659 (5)	16.646 (6)	20.967 (1)	-4.137 (1)	14.93 (1)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Tabel B. 3 Statistical values of *du* (diameter under bark) from compatible taper functions

Equation	Model fitting			Validation		
	SEE	MRES	MAE	RMSE	MRES	MAE
4.21	**0.923 (3)**	**-0.155 (5)**	**0.714 (1)**	**0.936 (4)**	**-0.183 (6)**	**0.714 (6)**
4.24	0.92 (2)	-0.157 (6)	0.717 (3)	0.914 (2)	-0.155 (4)	0.712 (4)
4.26	**0.917 (1)**	**-0.129 (3)**	**0.715 (2)**	0.929 (3)	-0.162 (5)	0.707 (5)
4.27	1.159 (6)	-0.015 (1)	0.919 (6)	1.179 (6)	0.057 (1)	0.934 (1)
4.28	1.017 (5)	-0.15 (4)	0.792 (5)	1.009 (5)	-0.086 (2)	0.796 (2)
4.29	0.924 (4)	-0.117 (2)	0.718 (4)	**0.908 (1)**	**-0.1 (3)**	**0.701 (3)**

*Note: * = the statistical value of d, hence we get comparison equally among functions.*
Ranking is shown in brackets. (The smallest rank was preferable).

Appendix C Coefficient of equations and the statistical values for Acacia mangium

Table C. 1 *Coefficient of equations and the statistical values of MTH (*Acacia mangium*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.65195			0.737 (19)	2.949 (20)	0.737 (1)	6039 (20)	0.912 (20)	3.253 (20)	2.529 (20)
2	Lundqvist–Korf (Schumacher Anamorphic II)	4.21016	0.24103		-0.039 (11)	1.916 (5)	1.426 (6)	4999 (5)	0.306 (11)	2.143 (11)	1.592 (5)
3	Schumacher/ Polymorphic I	3.366673			-0.015 (1)	2.081 (16)	1.696 (18)	5197 (16)	-0.069 (2)	2.25 (2)	1.763 (16)
4	Schumacher/ Polymorphic II	4.62222	0.38824		-0.169 (16)	1.561 (2)	1.206 (3)	4505 (2)	0.127 (4)	1.756 (4)	1.316 (2)
5	Schumacher /Polymorphic III	3.310755	0.965498		-0.169 (16)	2.056 (15)	1.667 (16)	5169 (15)	-0.197 (5)	2.225 (5)	1.749 (15)
6	Schumacher/ Polymorphic IV	1.190826	0.074999		-314.288 (23)	2.736 (19)	314.416 (23)	5859 (19)	0.562 (18)	3.117 (18)	2.31 (19)
7	Johnson–Schumacher	5.12279	1.38026		0.031 (9)	1.935 (11)	1.444 (12)	5023 (11)	0.308 (13)	2.18 (13)	1.621 (12)
8	von Bertalanffy–Richards Anamorphic	0.17214	1.04346		0.023 (4)	1.927 (8)	1.436 (9)	5013 (8)	0.304 (8)	2.161 (8)	1.607 (10)
9	**von Bertalanffy – Richards Polymorphic I**	**33.38764**	**0.215848**		**-0.048 (12)**	**1.315 (1)**	**1.022 (2)**	**4090 (1)**	**-0.006 (1)**	**1.468 (1)**	**1.081 (1)**
10	von Bertalanffy – Richards Polymorphic II	30.04882	0.16642		-0.016 (2)	1.584 (3)	1.224 (4)	4540 (3)	0.12 (3)	1.788 (3)	1.347 (3)
11	Weibull	0.159655	1.033702		0.023 (6)	1.928 (9)	1.437 (10)	5014 (9)	0.306 (10)	2.161 (10)	1.607 (9)
13	Monomolecular	1.008004	0.16164		0.02 (3)	1.925 (7)	1.434 (8)	5011 (7)	0.301 (6)	2.159 (6)	1.606 (8)
15	Gompertz	2.549372	0.497843		0.05 (13)	2.021 (14)	1.523 (15)	5128 (14)	0.325 (15)	2.286 (15)	1.705 (14)
16	Gompertz Polymorphic I	-0.11253	-0.046988		2.79 (21)	5.532 (22)	4.417 (21)	7559 (22)	3.168 (22)	5.58 (22)	4.421 (22)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
17	Logistic I	7.29385	0.85285		0.129 (15)	2.195 (17)	1.679 (17)	5327 (17)	0.396 (17)	2.481 (17)	1.859 (17)
18	Logistic II	1.39837	0.19914		0.031 (10)	1.931 (10)	1.439 (11)	5018 (10)	0.316 (14)	2.16 (14)	1.605 (7)
19	Logistic III	-2.006387	0.36896		0.321 (18)	2.598 (18)	2.038 (19)	5734 (18)	0.577 (19)	2.893 (19)	2.217 (18)
20	Hossfeld Anamorphic	0.149837	1.00976		1.386 (20)	3.938 (21)	2.72 (20)	6738 (21)	1.891 (21)	4.337 (21)	2.981 (21)
21	Hossfeld Polymorphic	1.2380296	0.0966413		-17.438 (22)	271.8 (23)	73.309 (22)	16960 (23)	-8.422 (23)	1624.404 (23)	145.74 (23)
22	Hossfeld IV	9.1587	1.07257		0.025 (8)	1.923 (6)	1.432 (7)	5008 (6)	0.306 (12)	2.156 (12)	1.603 (6)
23	Levakovic III	51.596725	0.47456		0.023 (5)	1.938 (12)	1.445 (13)	5026 (12)	0.305 (9)	2.175 (9)	1.618 (11)
24	Hyperbola I	0.134889	0.030969		0.064 (14)	1.983 (13)	1.493 (14)	5082 (13)	0.327 (16)	2.256 (16)	1.681 (13)
25	Hyperbola II	0.14634	0.11504	0.19284	0.024 (7)	1.909 (4)	1.42 (5)	4991 (4)	0.304 (7)	2.13 (7)	1.583 (4)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 2 *Coefficient of equations and the statistical values of Basal Area (*Acacia mangium*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	2.19878			0.483 (18)	3.447 (16)	2.465 (14)	6416 (16)	0.58 (18)	3.611 (18)	2.559 (15)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.41028	0.81588		0.305 (7)	3.391 (10)	2.438 (8)	6377 (10)	0.424 (8)	3.487 (8)	2.478 (7)
3	Schumacher/ Polymorphic I	3.389637			-0.112 (5)	2.229 (3)	1.694 (3)	5363 (3)	-0.143 (5)	2.523 (3)	1.948 (3)
4	**Schumacher/ Polymorphic II**	**3.49873**	**0.89706**		**-0.096 (4)**	**2.212 (1)**	**1.675 (1)**	**5346 (1)**	**-0.112 (4)**	**2.487 (1)**	**1.911 (1)**
5	Schumacher /Polymorphic III	3.409225	1.01174		-0.074 (3)	2.227 (2)	1.689 (2)	5362 (2)	-0.105 (2)	2.523 (2)	1.911 (1)
6	Schumacher/ Polymorphic IV	2.45081	-0.055598	-0.0556	0.748 (20)	3.31 (6)	153.994 (23)	6319 (6)	0.788 (20)	3.56 (15)	129.301 (23)
7	Johnson–Schumacher	2.76331	0.20605		0.308 (9)	3.39 (9)	2.439 (9)	6377 (9)	0.425 (9)	3.49 (9)	2.48 (9)
8	von Bertalanffy–Richards	0.57586	2.27624		0.332 (12)	3.423 (15)	2.478 (15)	6400 (15)	0.444 (12)	3.536 (14)	2.528 (14)
9	Anamorphic von Bertalanffy –Richards Polymorphic I	23.93822	0.42937		0.009 (1)	2.335 (4)	1.803 (4)	5477 (4)	-0.007 (1)	2.565 (4)	2.021 (4)
11	Weibull	0.181274	1.701013		0.364 (15)	3.472 (17)	2.535 (16)	6434 (17)	0.475 (15)	3.588 (16)	2.587 (16)
13	Monomolecular	1.19434	0.31652		0.309 (10)	3.402 (11)	2.449 (10)	6385 (11)	0.432 (11)	3.499 (10)	2.49 (10)
14	Richards	1.19836	0.34676	1.93218	0.309 (11)	3.404 (12)	2.45 (11)	6387 (12)	0.431 (10)	3.501 (11)	2.492 (11)
15	Gompertz	4.05638	0.80977		0.387 (16)	3.474 (18)	2.538 (17)	6435 (18)	0.486 (16)	3.608 (17)	2.597 (17)
16	Gompertz Polymorphic I	-0.094244	-0.033462		4.482 (23)	7.431 (23)	5.658 (22)	8271 (23)	4.615 (23)	7.374 (23)	5.592 (22)
17	Logistic I	18.38153	1.41121		0.588 (19)	3.66 (20)	2.747 (19)	6561 (20)	0.656 (19)	3.831 (20)	2.825 (19)
18	Logistic II	-2.13939	1.10044		0.471 (17)	3.552 (19)	2.629 (18)	6489 (19)	0.565 (17)	3.69 (19)	2.69 (18)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
19	Logistic III	-2.863162	0.549171		0.985 (21)	4.052 (21)	3.161 (20)	6807 (21)	0.998 (21)	4.26 (21)	3.278 (20)
20	Hossfeld Anamorphic	0.151991	1.292956		4.18 (22)	6.998 (22)	5.064 (21)	8126 (22)	4.421 (22)	6.944 (22)	4.974 (21)
21	Hossfeld Polymorphic	0.0422177	-2.0419543		-0.061 (2)	2.375 (5)	1.816 (5)	5518 (5)	-0.109 (3)	2.692 (5)	2.077 (5)
22	Hossfeld IV	6.34066	1.94794		0.335 (13)	3.409 (13)	2.462 (12)	6390 (13)	0.446 (13)	3.521 (12)	2.511 (12)
23	Levakovic III	6.75875	0.96745		0.338 (14)	3.411 (14)	2.463 (13)	6391 (14)	0.447 (14)	3.527 (13)	2.514 (13)
24	Hyperbola I	0.036055	0.071844		0.308 (8)	3.387 (7)	2.438 (7)	6374 (7)	0.422 (7)	3.479 (6)	2.478 (8)
25	Hyperbola II	0.02017*	0.08632	0.01574*	0.298 (6)	3.387 (7)	2.435 (6)	6375 (8)	0.415 (6)	3.479 (6)	2.472 (6)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 3 *Coefficient of equations and the statistical values of mortality of* Acacia mangium *(using all-possible interval)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.40708			-38.728 (19)	177.2 (19)	136 (17)	15926 (19)	-37.683 (19)	168.7 (17)	131.5 (17)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.053387	-1.333574		-4.119 (5)	132.5 (5)	95.9 (5)	15226 (5)	-6.818 (7)	139.1 (7)	94.9 (4)
3	Schumacher/ Polymorphic I	6.738704			-13.303 (18)	176.8 (18)	139 (19)	15921 (18)	-4.723 (6)	177.7 (19)	133.7 (19)
4	Schumacher/ Polymorphic II	11.26233	-0.05542		-10.997 (14)	150.5 (13)	114.2 (13)	15533 (13)	-11.87 (14)	145.6 (13)	107.6 (13)
5	Schumacher /Polymorphic III	6.768613	1.008305		-10.636 (13)	176.6 (17)	138.4 (18)	15920 (17)	-2.002 (1)	177.5 (18)	133.3 (18)
7	Johnson–Schumacher	-20.989	-17.574		-4.405 (8)	132.1 (2)	95.2 (2)	15218 (2)	-7.08 (10)	139.7 (10)	95.1 (6)
11	Weibull	4.64266	-0.89812		-4.983 (10)	134.3 (10)	97.8 (8)	15258 (10)	-7.602 (12)	139.1 (5)	94.9 (3)
12	Weibull Polymorphic I	0.042446	1.417925	-21.754	0.014 (1)	132.3 (3)	96.3 (7)	15222 (4)	-2.834 (2)	138.6 (4)	94.9 (2)
13	Monomolecular	0.66687	-0.03466*		-4.248 (6)	132 (1)	95.3 (3)	15217 (1)	-6.89 (8)	139.6 (9)	95.2 (7)
16	Gompertz Polymorphic I	0.0142006	-0.0005134		-2.321 (3)	133.9 (9)	99.4 (12)	15250 (9)	-3.852 (3)	137.2 (3)	96.1 (11)
17	Logistic I	0.38481	-0.2312		-4.286 (7)	132.3 (3)	95.5 (4)	15222 (3)	-6.993 (9)	139.3 (8)	95.7 (10)
20	Hossfeld Anamorphic	0.00003423	-1.53		-9.653 (12)	135.1 (12)	98.9 (11)	15272 (12)	-9.732 (13)	142.8 (11)	98.2 (12)
21	Hossfeld Polymorphic	0.2825	0.0001476		-11.141 (15)	150.6 (14)	114.3 (14)	15534 (14)	-11.929 (15)	145.6 (14)	107.7 (14)
22	Hossfeld IV	0.041804	-1.609544		-4.543 (9)	133 (6)	96.2 (6)	15235 (6)	-7.293 (11)	139.1 (6)	94.7 (1)
26	Anamorphic II	0.08592	1.2285		-8.772 (11)	134.5 (11)	95.2 (1)	15261 (11)	-12.226 (18)	143.9 (12)	95.5 (8)
27	Anamorphic III	-0.235249			-11.819 (16)	150.6 (14)	114.8 (15)	15534 (15)	-11.936 (16)	146.1 (15)	108.1 (15)
28	Anamorphic IV	-0.0335028			-12.464 (17)	150.9 (16)	115.2 (16)	15539 (16)	-12.148 (17)	146.6 (16)	108.5 (16)
29	Polymorphic I	-130.77492	0.08992		-1.984 (2)	133.6 (7)	98.3 (9)	15246 (8)	-4.397 (4)	137 (1)	95.5 (9)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
30	Exponential decay anamorphic	-0.100886			-2.401 (4)	133.6 (7)	98.6 (10)	15245 (7)	-4.444 (5)	137.1 (2)	95 (5)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 4 *Coefficient of equations and the statistical values of mortality of* Acacia mangium *(using two steps regression: 1 year interval data)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.30405			-41.897 (20)	129.6 (18)	98.5 (18)	6349 (18)	-48.643 (20)	130.8 (18)	99.8 (18)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.15875	-0.84603		-3.193 (9)	112.6 (4)	79.9 (10)	6208 (4)	-8.913 (9)	116.5 (3)	78.8 (5)
3	Schumacher/ Polymorphic I	6.82471			-26.746 (19)	147.8 (20)	113.5 (20)	6482 (20)	-26.19 (19)	152.2 (20)	113.7 (20)
4	Schumacher/ Polymorphic II	7.84197	-0.28982		-12.518 (15)	115.6 (13)	83.2 (14)	6235 (13)	-21.827 (15)	119.8 (13)	83.9 (14)
5	Schumacher /Polymorphic III	7.23784	1.067655		-10.133 (13)	139.1 (19)	102.2 (19)	6422 (19)	-9.132 (11)	143.4 (19)	101.9 (19)
7	Johnson–Schumacher	-196.54*	38.23*		-3.099 (7)	112.7 (5)	79.9 (8)	6209 (7)	-8.785 (7)	116.8 (6)	78.8 (7)
8	von Bertalanffy–Richards Anamorphic	-1.27362	-0.08473		-3.163 (8)	112.4 (3)	79.7 (5)	6207 (3)	-8.963 (10)	116.3 (2)	78.7 (3)
11	Weibull	3.0126	-0.7293		-4.665 (12)	113.2 (11)	81 (12)	6213 (11)	-10.246 (13)	116.8 (8)	79.2 (12)
12	Weibull Polymorphic I	-0.1243*	-0.6636*	-102.496	-0.02 (1)	111.2 (1)	78.3 (1)	6197 (1)	-7.449 (2)	115.1 (1)	78 (1)
13	Monomolecular	-4.58968	0.15738		-3.04 (4)	112.8 (8)	79.8 (6)	6210 (9)	-8.721 (4)	116.9 (9)	78.8 (4)
15	Gompertz	-2.87675	0.04629		-3.067 (5)	112.7 (5)	79.9 (7)	6209 (8)	-8.752 (5)	116.8 (7)	78.8 (6)
16	Gompertz Polymorphic I	0.012151	-0.004673		-3.401 (10)	113.2 (11)	79.5 (4)	6214 (12)	-8.91 (8)	117.7 (12)	79 (11)
17	Logistic I	-1.88515	-0.06485		-3.08 (6)	112.7 (5)	79.9 (9)	6209 (6)	-8.773 (6)	116.7 (5)	78.8 (8)
20	Hossfeld Anamorphic	0.00009582	-1.017		-10.448 (14)	116.8 (15)	82.3 (13)	6245 (15)	-14.059 (14)	121.1 (17)	82.4 (13)
21	Hossfeld Polymorphic	0.506	0.0004671		-13.031 (16)	115.8 (14)	83.5 (15)	6236 (14)	-22.995 (16)	120.4 (15)	84.7 (15)
22	Hossfeld IV	0.12991	-1.11814		-3.751 (11)	112.8 (8)	80.3 (11)	6210 (10)	-9.411 (12)	116.6 (4)	78.9 (10)
27	Anamorphic III	-0.219633			-18.544 (17)	118.5 (16)	87.4 (16)	6259 (16)	-24.64 (17)	120 (14)	84.9 (16)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
28	Anamorphic IV	-0.030877			-19.642 (18)	119.2 (17)	87.9 (17)	6264 (17)	-25.451 (18)	120.5 (16)	85.4 (17)
29	Polymorphic I	-973.16774*	0.06351		-0.266 (2)	112.3 (2)	78.4 (2)	6205 (2)	-6.858 (1)	117.6 (11)	78.6 (2)
30	**Exponential decay anamorphic**	**-0.118679**			**-2.513 (3)**	**112.8 (8)**	**79.2 (3)**	**6209 (5)**	**-8.232 (3)**	**117.6 (10)**	**78.9 (9)**

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 5 *Coefficient of equations and the statistical value of mortality of* Acacia mangium *(using longest interval)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.4311			0.065 (1)	183.7 (18)	146 (18)	2842 (18)	-3.868 (1)	195.3 (14)	147.8 (17)
2	Lundqvist–Korf (Schumacher Anamorphic II)	1.0143*	-0.2039*		-7.77 (13)	176.2 (7)	140.1 (6)	2825 (8)	-13.213 (12)	191.7 (8)	141.6 (3)
3	Schumacher/ Polymorphic I	6.71491			0.106 (2)	212.6 (20)	169.1 (20)	2904 (20)	12.244 (8)	229.3 (20)	176.8 (20)
4	Schumacher/ Polymorphic II	10.49881	-0.06862*		-4.389 (5)	176.1 (5)	140.1 (7)	2824 (6)	-11.526 (5)	191.3 (4)	142.3 (12)
5	Schumacher /Polymorphic III	6.59616	0.95318		-6.654 (9)	208.8 (19)	165.8 (19)	2897 (19)	7.551 (2)	221.8 (19)	168.6 (19)
7	Johnson–Schumacher	-3.505	3.297		-7.643 (12)	175.8 (3)	140.6 (12)	2826 (11)	-12.735 (10)	191.5 (6)	141.7 (5)
8	von Bertalanffy–Richards Anamorphic	-0.2639*	-0.1798		-8.927 (15)	181.3 (17)	139.5 (3)	2824 (4)	-14.731 (15)	192.2 (12)	141.7 (4)
11	Weibull	1.0356*	-0.3687		-7.162 (10)	176.3 (9)	140.3 (10)	2825 (10)	-12.47 (9)	191.5 (5)	141.6 (1)
12	Weibull Polymorphic I	0.000004354*	6.45	-229.6	0.347 (3)	163.5 (1)	128.1 (1)	2794 (1)	-18.974 (18)	205.2 (18)	153.7 (18)
13	Monomolecular	-1.03469	0.40592		-9.825 (17)	177.8 (13)	141 (15)	2829 (14)	-14.796 (16)	192.1 (11)	141.8 (8)
15	Gompertz	-0.76378	0.30471		-9.211 (16)	177.4 (12)	140.8 (13)	2828 (13)	-14.283 (14)	192 (10)	141.8 (7)
16	Gompertz Polymorphic I	0.010625	-0.013309		-4.697 (6)	173.4 (2)	138.7 (2)	2818 (2)	-11.544 (6)	192.7 (13)	142.7 (13)
17	Logistic I	-0.57092	0.20498		-8.654 (14)	177 (11)	140.6 (11)	2827 (12)	-13.827 (13)	191.9 (9)	141.8 (6)
20	Hossfeld Anamorphic	0.0002834*	-0.5173711		-15.536 (18)	181.2 (16)	142.6 (17)	2837 (17)	-15.236 (17)	197 (15)	145.6 (14)
21	Hossfeld Polymorphic	0.314627	0.000212*		-4.344 (4)	176 (4)	139.9 (5)	2824 (5)	-12.029 (7)	191 (2)	142 (10)
22	Hossfeld IV	0.9618	-0.4251**		-7.452 (11)	176.2 (7)	140.2 (8)	2825 (9)	-12.815 (11)	191.6 (7)	141.6 (2)
27	Anamorphic III	-0.23988			-4.934 (7)	176.1 (5)	140.9 (14)	2823 (3)	-9.945 (4)	191 (1)	142 (9)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
28	Anamorphic IV	-0.034234			-5.46 (8)	176.6 (10)	141.4 (16)	2825 (7)	-9.49 (3)	191.3 (3)	142.2 (11)
29	Polymorphic I	162.74983*	0.12323*		-25.304 (19)	181 (15)	140.2 (9)	2836 (16)	-32.142 (19)	200.5 (16)	145.7 (15)
30	Exponential decay anamorphic	-0.103897			-25.899 (20)	180.7 (14)	139.7 (4)	2835 (15)	-33.858 (20)	201.4 (17)	146.3 (16)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 6 *Coefficient of equations and the statistical values of Dmax (*Acacia mangium*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAB**	**AIC**	**MRES**	**RMSE**	**MAB**
1	Schumacher /Anamorphic I	1.34464			0.644 (20)	3.37 (19)	2.525 (19)	6361 (19)	0.423 (18)	2.945 (19)	10.68 (19)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.6875	0.3267		0.009 (4)	2.662 (8)	1.933 (6)	5793 (8)	-0.126 (9)	2.271 (8)	8.459 (6)
3	Schumacher/ Polymorphic I	3.526002			-0.059 (12)	2.647 (7)	2.001 (14)	5778 (7)	-0.493 (19)	2.259 (7)	9.08 (14)
4	Schumacher/ Polymorphic II	4.31687	0.4483		-0.05 (10)	2.241 (2)	1.657 (2)	5377 (2)	-0.334 (14)	1.854 (2)	7.112 (2)
5	Schumacher /Polymorphic III	3.493363	0.98114		-0.151 (16)	2.637 (6)	1.987 (13)	5770 (6)	-0.584 (20)	2.255 (6)	9.202 (16)
6	Schumacher/ Polymorphic IV	1.120781	0.023099		0.425 (19)	3.317 (18)	2.433 (18)	6324 (18)	0.26 (13)	2.91 (18)	10.455 (18)
7	Johnson– Schumacher	3.48837	1.09119		0.028 (8)	2.7 (11)	1.964 (10)	5827 (11)	-0.114 (4)	2.308 (10)	8.588 (8)
8	von Bertalanffy–Richards Anamorphic	0.2221	0.90206		0.007 (3)	2.7 (11)	1.964 (9)	5827 (12)	-0.126 (10)	2.312 (12)	8.628 (10)
9	**von Bertalanffy – Richards Polymorphic I**	**40.0433**	**0.19326**		**-0.051 (11)**	**2.129 (1)**	**1.59 (1)**	**5254 (1)**	**-0.4 (17)**	**1.777 (1)**	**6.864 (1)**
10	von Bertalanffy – Richards Polymorphic II	34.638076	-0.006663*		-0.062 (13)	2.329 (4)	1.726 (4)	5470 (4)	-0.355 (16)	1.951 (4)	7.557 (4)
11	Weibull	0.25603	0.91926		0.009 (5)	2.696 (10)	1.961 (8)	5824 (10)	-0.124 (7)	2.31 (11)	8.619 (9)
13	Monomolecular	0.97963	0.260712		0.004 (1)	2.709 (13)	1.973 (11)	5835 (13)	-0.133 (11)	2.318 (13)	8.646 (11)
15	Gompertz	2.10582	0.54337		0.062 (14)	2.834 (16)	2.088 (16)	5944 (16)	-0.084 (2)	2.449 (16)	9.141 (15)
16	Gompertz Polymorphic I	-0.081754	-0.030073		2.581 (22)	6.002 (22)	4.804 (22)	7755 (22)	2.573 (22)	5.853 (22)	22.699 (22)
17	Logistic I	4.91586	0.84185		0.157 (17)	2.998 (17)	2.236 (17)	6080 (17)	0.001 (1)	2.622 (17)	9.801 (17)
19	Logistic III	-1.602518	0.373038		0.363 (18)	3.382 (20)	2.584 (20)	6371 (20)	0.191 (12)	3.025 (20)	11.461 (20)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAB**	**AIC**	**MRES**	**RMSE**	**MAB**
20	Hossfeld Anamorphic	0.083879	0.990335		1.554 (21)	4.665 (21)	3.213 (21)	7147 (21)	1.678 (21)	4.128 (21)	13.867 (21)
21	Hossfeld Polymorphic	0.0226222	-1.1146112		-0.045 (9)	2.289 (3)	1.693 (3)	5428 (3)	-0.335 (15)	1.906 (3)	7.352 (3)
22	Hossfeld IV	5.02555	0.99516		0.013 (6)	2.682 (9)	1.949 (7)	5811 (9)	-0.122 (6)	2.293 (9)	8.544 (7)
23	Levakovic III	35.397475	0.403112		0.014 (7)	2.72 (14)	1.981 (12)	5845 (14)	-0.121 (5)	2.332 (14)	8.707 (12)
24	Hyperbola I	0.198218	0.059484		0.066 (15)	2.742 (15)	2.006 (15)	5865 (15)	-0.089 (3)	2.356 (15)	8.736 (13)
25	Hyperbola II	-0.10397*	0.44898	0.1474	0.005 (2)	2.626 (5)	1.906 (5)	5761 (5)	-0.124 (8)	2.235 (5)	8.305 (5)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 7 *Coefficient of equations and the statistical values of Dstd (*Acacia mangium*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAB**	**AIC**	**MRES**	**RMSE**	**MAB**
1	Schumacher /Anamorphic I	1.29404			0.147 (20)	0.707 (20)	0.539 (20)	2590 (19)	0.152 (20)	0.699 (19)	0.54 (20)
2	Lundqvist–Korf (Schumacher Anamorphic II)	4.21729	0.18116		0.014 (9)	0.565 (6)	0.404 (6)	2052 (6)	0.041 (12)	0.55 (6)	0.402 (6)
3	Schumacher/ Polymorphic I	1.742717			-0.018 (12)	0.57 (14)	0.443 (17)	2072 (14)	-0.044 (15)	0.553 (14)	0.421 (16)
4	Schumacher/ Polymorphic II	2.7299	0.3927		-0.011 (2)	0.489 (2)	0.36 (2)	1700 (2)	-0.008 (3)	0.468 (2)	0.347 (2)
5	Schumacher /Polymorphic III	1.70676	0.95774		-0.036 (17)	0.569 (13)	0.439 (16)	2067 (13)	-0.061 (17)	0.552 (11)	0.417 (15)
6	Schumacher/ Polymorphic IV	1.23773	0.03268*		0.141 (19)	0.706 (19)	0.536 (19)	2590 (20)	0.147 (19)	0.701 (20)	0.539 (19)
7	Johnson– Schumacher	4.51947	1.58283		0.015 (10)	0.567 (11)	0.405 (12)	2059 (11)	0.04 (11)	0.552 (13)	0.403 (12)
8	von Bertalanffy–Richards Anamorphic	0.13036	0.77993		0.014 (5)	0.566 (9)	0.404 (9)	2055 (9)	0.04 (8)	0.55 (9)	0.402 (9)
9	**von Bertalanffy – Richards Polymorphic I**	**7.93911**	**0.1335**		**-0.022 (14)**	**0.459 (1)**	**0.338 (1)**	**1547 (1)**	**-0.025 (4)**	**0.452 (1)**	**0.337 (1)**
11	Weibull	0.19249	0.81046		0.014 (6)	0.566 (8)	0.404 (8)	2054 (8)	0.04 (9)	0.55 (8)	0.4 (4)
12	Weibull Polymorphic I	3.84247	-0.1867	0.10537	-0.022 (14)	0.562 (4)	0.402 (4)	2037 (4)	0.001 (1)	0.546 (4)	0.402 (8)
13	Monomolecular	0.948118	0.205001		0.013 (3)	0.567 (12)	0.405 (11)	2060 (12)	0.039 (5)	0.552 (12)	0.403 (11)
15	Gompertz	1.99397	0.47027		0.017 (11)	0.575 (16)	0.411 (14)	2094 (16)	0.043 (13)	0.561 (16)	0.411 (14)
16	Gompertz Polymorphic I	-0.17652	-0.07452		0.764 (22)	1.313 (22)	1.017 (22)	4087 (22)	0.814 (22)	1.31 (22)	1.018 (22)
17	Logistic I	4.53098	0.74637		0.027 (16)	0.589 (17)	0.423 (15)	2150 (17)	0.051 (16)	0.576 (17)	0.425 (17)
19	Logistic III	-1.531975	0.342001		0.051 (18)	0.627 (18)	0.46 (18)	2301 (18)	0.075 (18)	0.615 (18)	0.464 (18)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAB**	**AIC**	**MRES**	**RMSE**	**MAB**
20	Hossfeld Anamorphic	0.49319	0.66021		0.453 (21)	1.005 (21)	0.72 (21)	3440 (21)	0.478 (21)	1.004 (21)	0.736 (21)
21	Hossfeld Polymorphic	0.12398	-1.05275		-0.01 (1)	0.491 (3)	0.362 (3)	1712 (3)	-0.005 (2)	0.479 (3)	0.353 (3)
22	Hossfeld IV	7.7386	0.83746		0.014 (8)	0.566 (7)	0.404 (7)	2053 (7)	0.04 (10)	0.55 (7)	0.402 (7)
23	Levakovic III	74.433443	0.361631		0.013 (4)	0.567 (10)	0.405 (10)	2059 (10)	0.04 (7)	0.551 (10)	0.402 (10)
24	Hyperbola I	0.218563	0.039568		0.02 (13)	0.571 (15)	0.408 (13)	2078 (15)	0.043 (14)	0.559 (15)	0.409 (13)
25	Hyperbola II	0.30163	0.12769	0.13481	0.014 (7)	0.564 (5)	0.403 (5)	2049 (5)	0.04 (6)	0.549 (5)	0.401 (5)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table C. 8 *Coefficient of equations and the statistical values of height-dbhob equations (*Acacia mangium*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		a	***b***	***c***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
4.30	Näslund	0.1288787	2.2050277		-0.006 (2)	1.986 (7)	1.502 (6)	139890 (1)	-0.036 (8)	1.999 (7)	1.513 (6)
4.31	Näslund-Schmidt	2.2189981	0.2069196		0.032 (9)	1.981 (6)	1.505 (7)	142216 (7)	0.001 (1)	1.998 (6)	1.517 (7)
4.32	Näslund 5	1.7554369	0.4784088		0.094 (13)	2.001 (10)	1.541 (10)	142889 (10)	0.062 (11)	2.019 (10)	1.555 (10)
4.33	Curtis	0.538576	-0.16443		-0.075 (12)	2.057 (14)	1.569 (13)	144760 (14)	-0.102 (14)	2.069 (13)	1.577 (13)
4.34	Schumacher	33.24958	13.59881		0.154 (16)	2.052 (12)	1.611 (14)	144598 (12)	0.12 (15)	2.07 (14)	1.625 (14)
4.35	Meyer	-48.07	-0.01552		-0.153 (15)	2.092 (15)	1.63 (15)	145919 (15)	-0.179 (16)	2.104 (15)	1.636 (15)
4.36	Power	0.56022	1.154141		-0.069 (11)	2.054 (13)	1.564 (12)	144665 (13)	-0.096 (13)	2.066 (12)	1.572 (11)
4.37	Michael Menten	0.8132009	-2.444355		6.751 (18)	8.986 (18)	7.117 (18)	244490 (18)	6.743 (18)	8.967 (18)	7.086 (18)
4.38	Wykoff	3.611476	16.279244		0.104 (14)	2.013 (11)	1.559 (11)	143307 (11)	0.071 (12)	2.031 (11)	1.573 (12)
4.39	Garcia	29.13148	-374.721		0 (1)	2.313 (17)	1.845 (17)	152699 (17)	-0.036 (7)	2.329 (17)	1.859 (17)
4.40	Prodan	-0.02071	1.6696396	-2.0538	-0.217 (17)	2.248 (16)	1.797 (16)	150761 (16)	-0.245 (17)	2.252 (16)	1.801 (16)
4.41	Logistic	19.858145	17.341035	0.24332	-0.016 (6)	1.967 (3)	1.477 (3)	141725 (4)	-0.045 (10)	1.984 (3)	1.487 (3)
4.42	Weibull	19.49	0.003353	2.099	-0.009 (4)	1.96 (1)	1.471 (1)	141504 (3)	-0.039 (9)	1.977 (1)	1.482 (1)
4.43	Gompertz	23.370396	3.914872	0.13044	0.007 (3)	1.96 (1)	1.472 (2)	141503 (2)	-0.023 (6)	1.978 (2)	1.483 (2)
4.44	Sibessen	0.054328	3.295409	0.17923	0.032 (10)	1.986 (7)	1.512 (8)	142378 (8)	0.002 (3)	2.003 (8)	1.523 (8)
4.45	Lundqvist-Korf	123.43699	7.95264	0.46808	0.028 (7)	1.989 (9)	1.515 (9)	142487 (9)	-0.002 (4)	2.006 (9)	1.526 (9)
4.46	Ratkowsky	50.8509	25.6753	4.0575	0.014 (5)	1.976 (5)	1.494 (5)	142049 (6)	-0.016 (5)	1.993 (5)	1.506 (5)
4.47	Hossfeld IV	32.83	0.005634	1.771	0.029 (8)	1.972 (4)	1.493 (4)	141925 (5)	-0.001 (2)	1.99 (4)	1.505 (4)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Appendix D Coefficient of equations and the statistical values for Acacia crassicarpa

Table D. 1 *Coefficient of equations and the statistical values of MTH (*Acacia crassicarpa*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.60663			0.471 (18)	2.561 (17)	1.988 (17)	3004 (17)	-0.18 (1)	2.813 (17)	2.077 (17)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.11673	0.59406		0.105 (13)	2.264 (13)	1.738 (10)	2848 (13)	-0.48 (7)	2.559 (14)	1.952 (5)
3	Schumacher/ Polymorphic I	3.464458			-0.01 (2)	1.672 (4)	1.357 (3)	2461 (4)	-0.354 (3)	1.67 (4)	1.329 (3)
4	Schumacher/ Polymorphic II	3.76836	0.70907		-0.007 (1)	1.536 (2)	1.222 (2)	2355 (2)	-0.371 (5)	1.634 (2)	1.323 (2)
5	Schumacher /Polymorphic III	3.439542	0.985505		-0.071 (4)	1.665 (3)	1.361 (4)	2458 (3)	-0.415 (6)	1.662 (3)	1.336 (4)
6	Schumacher/ Polymorphic IV	1.591476	0.002056*		-1047.818 (22)	2.563 (18)	1047.847 (22)	3006 (18)	-1606.812 (22)	5540.184 (22)	1606.929 (22)
7	Johnson–Schumacher	2.77462	0.54067		0.086 (5)	2.261 (9)	1.735 (6)	2847 (11)	-0.504 (15)	2.556 (13)	1.955 (10)
8	von Bertalanffy–Richards Anamorphic	0.43078	1.39492		0.099 (7)	2.258 (6)	1.738 (9)	2845 (7)	-0.502 (14)	2.547 (8)	1.955 (11)
9	**von Bertalanffy – Richards Polymorphic I**	**28.17395**	**0.41302**		**-0.022 (3)**	**1.363 (1)**	**1.08 (1)**	**2203 (1)**	**-0.327 (2)**	**1.445 (1)**	**1.178 (1)**
11	Weibull	0.27562	1.25533		0.098 (6)	2.261 (9)	1.743 (13)	2847 (10)	-0.507 (16)	2.547 (9)	1.959 (13)
13	Monomolecular	1.095302	0.336157		0.103 (10)	2.257 (5)	1.733 (5)	2844 (5)	-0.494 (12)	2.551 (12)	1.952 (7)
15	Gompertz	2.71232	0.68759		0.108 (15)	2.291 (15)	1.776 (15)	2864 (15)	-0.512 (18)	2.564 (15)	1.987 (15)
16	Gompertz Polymorphic I	-0.099185	-0.03971		3.374 (20)	6.647 (21)	5.385 (20)	4218 (21)	3.193 (20)	6.287 (19)	5.11 (20)
17	Logistic I	7.6589	1.0748		0.16 (16)	2.401 (16)	1.878 (16)	2923 (16)	-0.487 (10)	2.632 (16)	2.061 (16)
18	Logistic II	-0.58347	0.6595		0.103 (11)	2.275 (14)	1.76 (14)	2854 (14)	-0.51 (17)	2.551 (11)	1.973 (14)
19	Logistic III	-2.027683	0.441881		0.321 (17)	2.75 (19)	2.201 (18)	3095 (19)	-0.37 (4)	2.867 (18)	2.287 (18)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
20	Hossfeld Anamorphic	0.10951	1.30149		2.053 (19)	4.864 (20)	3.594 (19)	3821 (20)	0.644 (19)	8.196 (20)	4.028 (19)
21	Hossfeld Polymorphic	1.2270142	0.1171473		16.608 (21)	188.1 (22)	71.074 (21)	8471 (22)	-179.538 (21)	3662.86 (21)	277.633 (21)
22	Hossfeld IV	4.26132	1.41461		0.101 (9)	2.258 (6)	1.736 (7)	2845 (6)	-0.492 (11)	2.547 (10)	1.952 (6)
23	Levakovic III	12.42635	0.58912		0.1 (8)	2.261 (9)	1.741 (12)	2846 (9)	-0.502 (13)	2.547 (7)	1.956 (12)
24	Hyperbola I	0.119574	0.077608		0.105 (14)	2.26 (8)	1.739 (11)	2846 (8)	-0.482 (9)	2.543 (5)	1.955 (9)
25	Hyperbola II	0.116287	0.085806	0.007989*	0.103 (12)	2.261 (9)	1.738 (8)	2848 (12)	-0.481 (8)	2.543 (6)	1.954 (8)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 2 *Coefficient of equations and the statistical values of Basal Area (*Acacia crassicarpa*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.99103			0.491 (6)	3.519 (13)	2.542 (12)	3408 (12)	-0.245 (16)	3.404 (7)	2.46 (6)
2	Lundqvist–Korf (Schumacher Anamorphic II)	1.91796	1.09801		0.57 (10)	3.511 (11)	2.518 (10)	3407 (11)	-0.177 (14)	3.411 (8)	2.478 (7)
3	Schumacher/ Polymorphic I	3.37759			-0.06 (3)	2.348 (2)	1.842 (2)	2894 (2)	-0.045 (4)	2.464 (2)	1.917 (1)
4	**Schumacher/ Polymorphic II**	**3.3664**	**1.01289**		**-0.062 (4)**	**2.35 (3)**	**1.843 (3)**	**2895 (3)**	**-0.041 (3)**	**2.466 (3)**	**1.917 (2)**
5	Schumacher /Polymorphic III	3.4102	1.01899		-0.001 (1)	2.344 (1)	1.837 (1)	2892 (1)	0.017 (1)	2.46 (1)	1.922 (3)
6	Schumacher/ Polymorphic IV	2.500245	-0.091435		0.858 (20)	3.037 (6)	341.224 (23)	3222 (6)	0.288 (18)	3.193 (6)	424.349 (23)
7	Johnson– Schumacher	1.74377	-0.10498		0.581 (11)	3.508 (10)	2.514 (7)	3405 (10)	-0.17 (12)	3.414 (9)	2.481 (8)
8	von Bertalanffy– Richards Anamorphic	0.76697	2.59203		0.594 (13)	3.579 (16)	2.582 (15)	3431 (16)	-0.155 (11)	3.478 (14)	2.545 (14)
9	von Bertalanffy – Richards Polymorphic I	23.36347	0.46285		0.098 (5)	2.747 (5)	2.216 (5)	3094 (5)	0.295 (19)	2.79 (5)	2.236 (5)
11	Weibull	0.23588	1.745251		0.614 (16)	3.644 (18)	2.668 (17)	3454 (18)	-0.137 (10)	3.544 (16)	2.607 (15)
13	Monomolecular	1.34479	0.49592		0.555 (9)	3.517 (12)	2.525 (11)	3409 (13)	-0.219 (15)	3.457 (13)	2.505 (10)
14	Richards	1.20258	0.22794	2.65805	0.547 (7)	3.494 (8)	2.515 (8)	3401 (8)	-0.414 (20)	4.349 (21)	2.671 (17)
15	Gompertz	4.16337	0.97146		0.647 (17)	3.63 (17)	2.638 (16)	3449 (17)	-0.101 (7)	3.535 (15)	2.611 (16)
16	Gompertz Polymorphic I	-0.08526	-0.02327*		4.38 (23)	7.502 (23)	5.753 (22)	4372 (23)	4.272 (23)	7.429 (23)	5.654 (22)
17	Logistic I	16.41887	1.55068		0.814 (19)	3.79 (20)	2.821 (19)	3504 (20)	0.073 (6)	3.703 (19)	2.793 (19)
18	Logistic II	-2.10984	1.2661		0.708 (18)	3.707 (19)	2.735 (18)	3475 (19)	-0.036 (2)	3.608 (18)	2.683 (18)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
19	Logistic III	-2.723969	0.598161		1.184 (21)	4.098 (21)	3.149 (20)	3603 (21)	0.471 (21)	4.016 (20)	3.114 (20)
20	Hossfeld Anamorphic	0.12745	1.57479		3.887 (22)	6.681 (22)	4.847 (21)	4225 (22)	3.397 (22)	6.13 (22)	4.36 (21)
21	Hossfeld Polymorphic	0.0442381	-2.1309081		-0.027 (2)	2.486 (4)	1.982 (4)	2967 (4)	-0.066 (5)	2.667 (4)	2.077 (4)
22	Hossfeld IV	4.49018	2.13085		0.603 (14)	3.559 (15)	2.557 (14)	3424 (15)	-0.136 (9)	3.448 (12)	2.515 (11)
23	Levakovic III	3.29939	1.17716		0.612 (15)	3.551 (14)	2.547 (13)	3421 (14)	-0.132 (8)	3.445 (11)	2.517 (12)
24	Hyperbola I	-0.04379	0.17886		0.586 (12)	3.505 (9)	2.512 (6)	3404 (9)	-0.176 (13)	3.43 (10)	2.488 (9)
25	Hyperbola II	-0.378521*	0.474203	0.044351	0.552 (8)	3.491 (7)	2.516 (9)	3400 (7)	-0.254 (17)	3.578 (17)	2.527 (13)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 3 *Coefficient of equations and the statistical values of mortality of* Acacia crassicarpa *(using all-possible interval)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.588			-59.393 (19)	252.9 (17)	202.4 (17)	8846 (17)	-67.985 (19)	258 (17)	208.1 (17)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.14551	-1.03553		2.745 (9)	178.2 (6)	137.5 (11)	8401 (6)	-9.261 (8)	206 (10)	158.3 (10)
3	Schumacher/ Polymorphic I	6.89538			-28.369 (18)	273.7 (18)	221.6 (19)	8946 (18)	-37.508 (18)	261.1 (19)	210.2 (19)
4	Schumacher/ Polymorphic II	13.37294	-0.05665**		-13.511 (13)	203.7 (13)	159.7 (13)	8572 (13)	-22.841 (12)	217.8 (14)	171.8 (13)
5	Schumacher /Polymorphic III	6.90681	1.00307		-27.17 (17)	273.9 (19)	221.2 (18)	8948 (19)	-36.294 (17)	260.9 (18)	209.8 (18)
7	Johnson– Schumacher	-48318.7*	554.4*		2.636 (6)	178.2 (6)	137.5 (9)	8402 (8)	-9.227 (5)	205.5 (7)	158.2 (6)
11	Weibull	3.94592	-1.02827		3.381 (11)	176 (1)	135.8 (1)	8386 (1)	-7.746 (1)	204.2 (1)	156.9 (1)
12	Weibull Polymorphic I	0.14787	1.03314	6.96066*	1.488 (3)	178.3 (11)	137.4 (4)	8403 (11)	-10.541 (9)	206 (11)	158.2 (8)
13	Monomolecular	-23.29639*	0.16636		2.596 (4)	178.2 (6)	137.5 (6)	8402 (7)	-9.176 (3)	205.2 (4)	158.1 (3)
15	Gompertz	-42.019591*	0.003743*		2.635 (5)	178.2 (6)	137.5 (8)	8402 (8)	-9.226 (4)	205.5 (6)	158.2 (5)
16	Gompertz Polymorphic I	0.0218126	0.0008428*		-0.436 (2)	177.3 (2)	137 (3)	8395 (3)	-12.272 (11)	204.7 (2)	158.1 (4)
17	Logistic I	28.22679*	-0.15933		2.687 (8)	178.2 (6)	137.5 (7)	8402 (10)	-9.245 (7)	205.7 (9)	158.3 (9)
20	Hossfeld Anamorphic	0.000041	-1.543		-12.915 (12)	180.2 (12)	140.5 (12)	8416 (12)	-26.968 (15)	208.8 (12)	162.6 (12)
21	Hossfeld Polymorphic	0.3806	0.00006358*		-14.727 (14)	204 (14)	160.3 (14)	8573 (15)	-23.813 (13)	217.8 (13)	172.2 (14)
22	Hossfeld IV	0.09885	-1.46726		3.312 (10)	177.3 (2)	136.8 (2)	8395 (2)	-8.329 (2)	205.2 (5)	157.7 (2)
27	Anamorphic III	-0.342455			-16.636 (15)	204 (14)	161.1 (15)	8573 (14)	-26.222 (14)	217.9 (15)	172.8 (15)
28	Anamorphic IV	-0.046695			-19.611 (16)	205.2 (16)	162.7 (16)	8580 (16)	-29.363 (16)	218.5 (16)	173.7 (16)
29	Polymorphic I	156.73361	0.17565		0.392 (1)	177.8 (4)	137.5 (5)	8398 (4)	-10.906 (10)	204.7 (3)	158.4 (11)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
30	Exponential decay anamorphic	0.155702			2.659 (7)	178.1 (5)	137.5 (10)	8400 (5)	-9.241 (6)	205.6 (8)	158.2 (7)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 4 *Coefficient of equations and the statistical values of mortality of* Acacia crassicarpa *(using two steps regression: 1 year interval data)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.37051			-83.835 (18)	200.2 (17)	155.6 (17)	3658 (17)	-107.255 (18)	208.9 (17)	164.2 (17)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.10276	-1.26592		1.71 (6)	138.6 (3)	107.3 (7)	3459 (3)	-20.584 (3)	152.5 (6)	118.9 (9)
3	Schumacher/ Polymorphic I	7.10157			-63.762 (17)	229.4 (18)	187.8 (18)	3732 (18)	-92.419 (17)	220.3 (18)	178.2 (18)
4	Schumacher/ Polymorphic II	9.15062	-0.16193		-29.925 (13)	167.1 (12)	126.2 (12)	3561 (12)	-51.381 (13)	174 (12)	132.6 (12)
5	Schumacher /Polymorphic III	8.08075	1.15077		-16.999 (12)	180.7 (16)	141.1 (16)	3603 (16)	-45.518 (12)	179.9 (16)	141.9 (16)
7	Johnson–Schumacher	-128.87*	-30.75**		0.615 (4)	139.6 (6)	107.4 (10)	3463 (6)	-21.812 (5)	152.4 (5)	118.6 (7)
11	**Weibull**	**4.41123**	**-1.13866**		**2.334 (8)**	**134.9 (1)**	**104.8 (1)**	**3444 (1)**	**-19.551 (2)**	**149.7 (1)**	**116.5 (1)**
12	Weibull Polymorphic I	0.11096	1.28196	29.56611*	0.083 (2)	138.8 (4)	107.1 (6)	3460 (4)	-22.191 (6)	152.5 (7)	118.5 (6)
13	Monomolecular	3.51604*	0.10162		0.101 (3)	139.8 (7)	107.4 (8)	3464 (7)	-22.365 (7)	152.5 (8)	118.5 (5)
16	Gompertz Polymorphic I	0.006093*	-0.015628*		-4.149 (10)	140.8 (10)	107 (5)	3467 (10)	-27.006 (10)	152.7 (10)	119 (10)
17	Logistic I	0.89696**	-0.25901		1.479 (5)	139.2 (5)	107.5 (11)	3461 (5)	-20.874 (4)	152.5 (9)	118.8 (8)
20	Hossfeld Anamorphic	0.0000324	-1.709		-13.096 (11)	141.2 (11)	107.4 (9)	3469 (11)	-38.413 (11)	157.4 (11)	120.6 (11)
21	Hossfeld Polymorphic	0.4292	0.0002025		-31.555 (14)	167.6 (13)	126.8 (13)	3562 (13)	-52.371 (14)	174.2 (13)	132.8 (13)
22	Hossfeld IV	0.06997	-1.72442		3.047 (9)	137 (2)	106.2 (2)	3452 (2)	-19.018 (1)	151.2 (2)	118 (4)
27	Anamorphic III	-0.27586			-38.902 (15)	169.2 (14)	128.6 (14)	3566 (14)	-62.016 (15)	176.2 (14)	133.8 (14)
28	Anamorphic IV	-0.036913			-41.611 (16)	170.3 (15)	129.8 (15)	3570 (15)	-65.06 (16)	177.2 (15)	134.6 (15)
29	Polymorphic I	-155.13466*	0.14506		-0.082 (1)	140.4 (9)	106.8 (4)	3466 (9)	-22.565 (8)	151.8 (3)	117.4 (3)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
30	Exponential decay anamorphic	0.159647			-1.996 (7)	140.3 (8)	106.7 (3)	3464 (8)	-24.452 (9)	151.8 (4)	117.2 (2)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 5 *Coefficient of equations and the statistical values of mortality of* Acacia crassicarpa *(using longest interval)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.61726			0.692 (1)	261.6 (17)	213.6 (17)	1330 (17)	-43.803 (17)	283.1 (18)	232.1 (18)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.2353*	-0.7864		3.7 (10)	236 (7)	190.3 (8)	1312 (10)	-31.173 (7)	268.6 (12)	213.2 (11)
3	Schumacher/ Polymorphic I	6.9002			-4.634 (15)	327.9 (19)	278.8 (19)	1373 (19)	-53.614 (18)	302.4 (19)	245.1 (19)
4	Schumacher/ Polymorphic II	-28.335867*	0.009468*		4.84 (16)	243.8 (16)	195.7 (14)	1318 (16)	-34.701 (12)	269.5 (14)	220.7 (14)
5	Schumacher /Polymorphic III	6.37422	0.74788		-11.297 (18)	293.3 (18)	242.1 (18)	1353 (18)	-42.348 (16)	282.4 (17)	228.9 (17)
7	Johnson–Schumacher	-90.98*	21.74*		3.728 (12)	236 (7)	190.2 (6)	1312 (8)	-30.783 (5)	268.2 (9)	213 (9)
11	Weibull	3.2232	-0.9059		3.575 (8)	235.8 (4)	189.7 (1)	1312 (4)	-29.791 (1)	267.4 (4)	212.4 (3)
12	Weibull Polymorphic I	0.3458*	0.6429*	71.6976*	1.753 (5)	237.1 (11)	190.6 (9)	1314 (12)	-32.802 (9)	268.1 (7)	213 (7)
13	Monomolecular	-3.639*	0.2329		3.765 (14)	235.9 (5)	190.1 (3)	1312 (5)	-30.475 (2)	268 (5)	212.9 (6)
15	Gompertz	-2.32383*	0.08189*		3.742 (13)	236 (7)	190.2 (5)	1312 (7)	-30.718 (4)	268.2 (8)	213 (8)
16	Gompertz Polymorphic I	0.01944	-0.005101*		1.566 (2)	234.2 (1)	189.8 (2)	1310 (1)	-34.578 (11)	265.9 (2)	212 (2)
17	Logistic I	-1.51378*	-0.06821*		3.718 (11)	236 (7)	190.2 (7)	1312 (9)	-30.939 (6)	268.4 (10)	213.1 (10)
20	Hossfeld Anamorphic	0.00004235	-1.516		-17.18 (19)	238.3 (12)	195.6 (13)	1314 (11)	-54.447 (19)	263.9 (1)	214.5 (12)
21	Hossfeld Polymorphic	0.3012	-0.00007679*		3.214 (7)	243.5 (15)	196.5 (15)	1318 (15)	-37.254 (14)	268.8 (13)	220.8 (15)
22	Hossfeld IV	0.1604*	-1.2034		3.649 (9)	235.9 (5)	190.1 (4)	1312 (6)	-30.555 (3)	268.1 (6)	212.9 (5)
27	Anamorphic III	-0.3377			5.496 (17)	242.5 (13)	195.3 (12)	1316 (13)	-33.834 (10)	269.9 (15)	220.6 (13)
28	Anamorphic IV	-0.04604			2.114 (6)	242.9 (14)	197.5 (16)	1316 (14)	-38.2 (15)	268.5 (11)	221.1 (16)
29	Polymorphic I	243.7198*	0.1859		-1.654 (4)	235.1 (2)	191.6 (11)	1311 (3)	-35.496 (13)	266 (3)	211.2 (1)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
30	Exponential decay anamorphic	0.153855			1.644 (3)	235.5 (3)	191.5 (10)	1310 (2)	-32.127 (8)	270.9 (16)	212.4 (4)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 6 *Coefficient of equations and the statistical values of Dmax of (*Acacia crassicarpa*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.34638			0.715 (18)	3.293 (18)	2.342 (18)	3324 (18)	0.25 (15)	2.45 (17)	1.915 (16)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.1089	0.4551		0.315 (11)	2.99 (7)	1.886 (6)	3202 (7)	-0.061 (5)	2.039 (6)	1.561 (4)
3	Schumacher/ Polymorphic I	3.461782			0.018 (3)	2.312 (5)	1.725 (5)	2874 (5)	-0.229 (14)	2.248 (15)	1.782 (15)
4	Schumacher/ Polymorphic II	3.8543	0.62124		0.042 (4)	2.132 (3)	1.521 (2)	2772 (3)	-0.222 (12)	1.99 (1)	1.569 (9)
5	Schumacher /Polymorphic III	3.41422	0.972313		-0.104 (5)	2.292 (4)	1.698 (4)	2864 (4)	-0.35 (17)	2.219 (14)	1.764 (14)
6	Schumacher/ Polymorphic IV	1.580221	-0.024935		0.856 (19)	3.21 (17)	2.456 (19)	3293 (17)	0.388 (18)	2.671 (18)	2.099 (18)
7	Johnson–Schumacher	2.8199	0.7896		0.328 (13)	3.014 (9)	1.898 (11)	3212 (10)	-0.054 (4)	2.037 (3)	1.558 (1)
8	von Bertalanffy–Richards Anamorphic	0.31685	1.0122		0.31 (8)	3.016 (11)	1.896 (9)	3213 (11)	-0.072 (9)	2.04 (7)	1.562 (5)
9	**von Bertalanffy – Richards Polymorphic I**	**32.38249**	**0.2898**		**-0.011 (2)**	**2.011 (1)**	**1.48 (1)**	**2698 (1)**	**-0.225 (13)**	**2.073 (12)**	**1.649 (12)**
10	von Bertalanffy – Richards Polymorphic II	29.0236	0.15491		0.002 (1)	2.124 (2)	1.57 (3)	2767 (2)	-0.338 (16)	2.014 (2)	1.591 (10)
11	Weibull	0.31006	1.00494		0.311 (9)	3.016 (11)	1.896 (8)	3213 (12)	-0.069 (8)	2.041 (8)	1.563 (6)
13	Monomolecular	1.00981	0.32327		0.306 (7)	3.014 (9)	1.897 (10)	3212 (9)	-0.08 (11)	2.038 (5)	1.56 (3)
15	Gompertz	2.15019	0.60417		0.336 (14)	3.089 (15)	1.956 (14)	3243 (15)	-0.063 (7)	2.073 (11)	1.608 (11)
16	Gompertz Polymorphic I	-0.079378	-0.015665		2.54 (20)	5.967 (20)	4.604 (20)	4081 (20)	2.516 (20)	5.416 (19)	4.337 (20)
17	Logistic I	4.96542	0.90148		0.381 (16)	3.179 (16)	2.055 (16)	3280 (16)	-0.033 (2)	2.148 (13)	1.698 (13)
19	Logistic III	-1.597352	0.388965		0.487 (17)	3.392 (19)	2.302 (17)	3362 (19)	0.038 (3)	2.395 (16)	1.927 (17)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
20	Hossfeld Anamorphic	0.130065	0.963371		-6.789 (21)	51.26 (21)	13.288 (21)	6817 (21)	3.537 (21)	240.974 (21)	23.681 (21)
21	Hossfeld Polymorphic	1.303916	0.109535		9.122 (22)	167.6 (22)	69.436 (22)	8323 (22)	-19.339 (22)	2253.384 (22)	261.103 (22)
22	Hossfeld IV	3.87802	1.11904		0.316 (12)	3.007 (8)	1.89 (7)	3209 (8)	-0.061 (6)	2.038 (4)	1.558 (2)
23	Levakovic III	21.05221	0.43679		0.312 (10)	3.026 (13)	1.903 (12)	3217 (13)	-0.072 (10)	2.043 (9)	1.566 (7)
24	Hyperbola I	0.185811	0.084128		0.376 (15)	3.057 (14)	1.93 (13)	3230 (14)	-0.011 (1)	2.049 (10)	1.568 (8)
25	Hyperbola II	-376.1*	169.4*	-146.7*	0.272 (6)	2.877 (6)	1.973 (15)	3154 (6)	-1.031 (19)	9.696 (20)	2.674 (19)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 7 *Coefficient of equations and the statistical values of Dstd (*Acacia crassicarpa*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.3254			0.153 (19)	0.722 (18)	0.55 (19)	1394 (18)	0 (1)	0.594 (19)	0.465 (19)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.36573	0.38028		0.063 (14)	0.651 (11)	0.484 (13)	1263 (12)	-0.068 (4)	0.529 (5)	0.399 (3)
3	Schumacher/ Polymorphic I	1.8279			0.004 (1)	0.632 (5)	0.5 (16)	1224 (5)	-0.164 (18)	0.571 (16)	0.445 (17)
4	Schumacher/ Polymorphic II	2.54652	0.47275		0.018 (5)	0.577 (3)	0.45 (2)	1110 (3)	-0.129 (14)	0.506 (1)	0.387 (1)
5	Schumacher /Polymorphic III	1.76091	0.92517		-0.03 (6)	0.626 (4)	0.499 (15)	1212 (4)	-0.198 (19)	0.567 (15)	0.445 (15)
6	Schumacher/ Polymorphic IV	1.31363	0.006346*		0.152 (18)	0.723 (19)	0.549 (18)	1396 (19)	-0.001 (2)	0.594 (18)	0.464 (18)
7	Johnson–Schumacher	3.2094	0.9899		0.062 (13)	0.652 (13)	0.483 (11)	1264 (13)	-0.069 (7)	0.531 (7)	0.4 (6)
8	von Bertalanffy–Richards Anamorphic	0.28127	0.95785		0.062 (9)	0.651 (7)	0.482 (8)	1263 (7)	-0.071 (9)	0.531 (9)	0.4 (7)
9	**von Bertalanffy – Richards Polymorphic I**	**7.53374**	**0.18659**		**0.007 (2)**	**0.571 (1)**	**0.453 (3)**	**1096 (1)**	**-0.147 (16)**	**0.526 (3)**	**0.4 (9)**
11	Weibull	0.2986	0.96876		0.062 (10)	0.651 (7)	0.482 (9)	1263 (8)	-0.071 (8)	0.531 (8)	0.4 (5)
12	Weibull Polymorphic I	1.91873	-0.41729	0.23477	-0.015 (3)	0.635 (6)	0.477 (4)	1232 (6)	-0.151 (17)	0.529 (4)	0.405 (12)
13	Monomolecular	0.989	0.29472		0.061 (8)	0.651 (9)	0.482 (7)	1263 (9)	-0.071 (10)	0.531 (10)	0.4 (8)
15	Gompertz	2.09816	0.57389		0.062 (11)	0.655 (14)	0.482 (5)	1271 (14)	-0.075 (12)	0.54 (13)	0.407 (13)
16	Gompertz Polymorphic I	-0.15936	-0.03568*		0.798 (21)	1.433 (21)	1.132 (21)	2267 (21)	0.74 (21)	1.249 (21)	0.992 (21)
17	Logistic I	4.83376	0.86884		0.066 (15)	0.663 (16)	0.485 (14)	1287 (16)	-0.075 (13)	0.552 (14)	0.418 (14)
19	Logistic III	-1.586614	0.38177		0.08 (17)	0.69 (17)	0.508 (17)	1336 (17)	-0.069 (5)	0.587 (17)	0.445 (16)
20	Hossfeld Anamorphic	0.333002	2.100753		0.658 (20)	1.166 (20)	0.871 (20)	2004 (20)	0.487 (20)	0.935 (20)	0.746 (20)
21	Hossfeld Polymorphic	0.128116	-1.149594		0.016 (4)	0.574 (2)	0.448 (1)	1103 (2)	-0.136 (15)	0.509 (2)	0.391 (2)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
22	Hossfeld IV	4.25405	1.0499		0.062 (12)	0.651 (11)	0.483 (12)	1263 (11)	-0.069 (6)	0.53 (6)	0.399 (4)
23	Levakovic III	25.68669	0.41707		0.061 (7)	0.651 (10)	0.482 (6)	1263 (10)	-0.072 (11)	0.532 (11)	0.401 (10)
24	Hyperbola I	0.204126	0.064639		0.067 (16)	0.656 (15)	0.483 (10)	1272 (15)	-0.067 (3)	0.534 (12)	0.403 (11)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table D. 8 *Coefficient of equations and the statistical values of height-dbhob equations (*Acacia crassicarpa*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		a	***b***	***c***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
4.30	**Naslund**	**0.1534813**	**1.4573949**		**0.04 (8)**	**2.482 (5)**	**1.876 (4)**	**56749 (5)**	**0.089 (12)**	**2.506 (5)**	**1.898 (4)**
4.31	Naslund-Schmidt	1.4793277	0.2317869		0.065 (11)	2.489 (9)	1.883 (5)	56824 (9)	0.113 (13)	2.511 (7)	1.905 (5)
4.32	Naslund 5	1.1807396	0.4961911		0.112 (14)	2.52 (11)	1.919 (10)	57126 (11)	0.161 (16)	2.537 (11)	1.94 (10)
4.33	Curtis	1.288415	0.077127		-0.06 (10)	2.531 (13)	1.958 (13)	57231 (13)	-0.012 (2)	2.574 (13)	1.982 (13)
4.34	Schumacher	29.48488	9.31485		0.157 (16)	2.572 (15)	1.981 (15)	57624 (15)	0.206 (17)	2.584 (15)	2.003 (15)
4.35	Meyer	72.28	0.01622		-0.088 (12)	2.514 (10)	1.946 (12)	57066 (10)	-0.04 (4)	2.552 (12)	1.969 (12)
4.36	Power	1.248547	0.932778		-0.056 (9)	2.533 (14)	1.959 (14)	57246 (14)	-0.008 (1)	2.576 (14)	1.983 (14)
4.37	Michael Menten	1.600076	-1.745945		7.293 (18)	10.01 (18)	7.89 (18)	90740 (18)	7.356 (18)	10.096 (18)	7.949 (18)
4.38	Wykoff	3.487358	11.539689		0.099 (13)	2.52 (11)	1.919 (11)	57126 (12)	0.148 (15)	2.536 (10)	1.941 (11)
4.39	Garcia	31.1186	-364.2159		0 (1)	2.667 (17)	2.103 (17)	58506 (17)	0.047 (7)	2.681 (17)	2.115 (17)
4.40	Prodan	0.0018585	0.9822878	-0.6282	-0.113 (15)	2.575 (16)	2.022 (16)	57651 (16)	-0.069 (10)	2.625 (16)	2.048 (16)
4.41	Logistic	20.595118	10.491014	0.24839	-0.024 (7)	2.486 (7)	1.893 (8)	56792 (7)	0.027 (3)	2.503 (4)	1.915 (8)
4.42	Weibull	21.47	0.01391	1.639	-0.008 (3)	2.469 (1)	1.867 (2)	56629 (3)	0.042 (5)	2.492 (2)	1.888 (2)
4.43	Gompertz	23.158219	3.125301	0.14436	-0.004 (2)	2.469 (1)	1.865 (1)	56626 (2)	0.046 (6)	2.49 (1)	1.886 (1)
4.44	Sibessen	0.346389	2.16886	0.15859	0.022 (6)	2.484 (6)	1.885 (6)	56775 (6)	0.07 (11)	2.512 (8)	1.908 (6)
4.45	Lundqvist-Korf	129.52172	5.98736	0.3864	0.019 (5)	2.486 (7)	1.888 (7)	56794 (8)	0.067 (8)	2.515 (9)	1.911 (7)
4.46	Ratkowsky	39.7287	16.5879	3.268	-0.183 (17)	2.469 (1)	1.895 (9)	50863 (1)	-0.134 (14)	2.506 (6)	1.916 (9)
4.47	Hossfeld IV	35.71	0.01673	1.431	0.019 (4)	2.476 (4)	1.871 (3)	56692 (4)	0.067 (9)	2.501 (3)	1.893 (3)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Appendix E Coefficient of equations and the statistical values for Eucalyptus pellita

Table E. 1 *Coefficient of equations and the statistical values of MTH (*Eucalyptus pellita*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.41392			0.565 (18)	2.818 (17)	2.199 (17)	4505 (17)	0.887 (18)	3.042 (17)	2.359 (17)
2	Lundqvist–Korf (Schumacher Anamorphic II)	1.962	0.55584		0.248 (13)	2.596 (8)	1.985 (13)	4356 (8)	0.552 (12)	2.961 (16)	2.24 (16)
3	Schumacher/ Polymorphic I	3.414873			0.026 (2)	2.178 (4)	1.748 (3)	4033 (4)	0.191 (3)	2.18 (3)	1.693 (4)
4	Schumacher/ Polymorphic II	3.70579	0.69961		0.056 (3)	2.093 (2)	1.628 (2)	3961 (2)	0.247 (4)	2.214 (4)	1.691 (3)
5	Schumacher /Polymorphic III	3.371759	0.974559		-0.075 (4)	2.162 (3)	1.748 (4)	4021 (3)	0.089 (1)	2.162 (2)	1.676 (2)
6	Schumacher/ Polymorphic IV	1.370886	0.005306*		-1829.323 (22)	2.818 (17)	1829.835 (22)	4506 (18)	-2017.528 (22)	8166.518 (22)	2018.095 (22)
7	Johnson–Schumacher	2.57748	0.59496		0.246 (11)	2.595 (5)	1.983 (7)	4355 (7)	0.55 (6)	2.948 (14)	2.223 (14)
8	von Bertalanffy–Richards Anamorphic	0.41608	1.21761		0.244 (6)	2.596 (8)	1.983 (8)	4356 (9)	0.55 (9)	2.938 (9)	2.21 (10)
9	**von Bertalanffy – Richards Polymorphic I**	**27.3124**	**0.41712**		**0.005 (1)**	**2.013 (1)**	**1.571 (1)**	**3889 (1)**	**0.153 (2)**	**2.121 (1)**	**1.612 (1)**
11	Weibull	0.322538	1.144369		0.243 (5)	2.598 (10)	1.984 (9)	4357 (10)	0.55 (8)	2.937 (8)	2.209 (8)
13	Monomolecular	1.06122	0.36167		0.244 (8)	2.595 (5)	1.982 (5)	4355 (5)	0.551 (11)	2.942 (11)	2.216 (12)
15	Gompertz	2.41132	0.68759		0.25 (14)	2.619 (15)	2 (15)	4372 (15)	0.559 (15)	2.925 (5)	2.192 (5)
16	Gompertz Polymorphic I	-0.103654	-0.03956		2.664 (20)	5.888 (21)	4.891 (20)	5858 (21)	2.822 (20)	6.496 (20)	5.403 (20)
17	Logistic I	6.09053	1.04304		0.283 (16)	2.679 (16)	2.052 (16)	4414 (16)	0.595 (16)	2.942 (12)	2.21 (9)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
18	Logistic II	0.10535	0.5612		0.245 (9)	2.602 (14)	1.987 (14)	4360 (14)	0.552 (14)	2.936 (7)	2.206 (7)
19	Logistic III	-1.81029	0.44131		0.396 (17)	2.891 (19)	2.248 (18)	4553 (19)	0.713 (17)	3.081 (18)	2.359 (18)
20	Hossfeld Anamorphic	0.093958	1.237402		1.863 (19)	4.424 (20)	3.263 (19)	5334 (20)	2.12 (19)	5.619 (19)	3.983 (19)
21	Hossfeld Polymorphic	1.1442121	0.1263323		5.79 (21)	237.3 (22)	78.75 (21)	12637 (22)	38.245 (21)	767.048 (21)	100.651 (21)
22	Hossfeld IV	3.60193	1.2908		0.245 (10)	2.595 (5)	1.982 (6)	4355 (6)	0.55 (7)	2.945 (13)	2.22 (13)
23	Levakovic III	13.18743	0.51515		0.244 (7)	2.598 (10)	1.984 (12)	4357 (11)	0.551 (10)	2.934 (6)	2.206 (6)
24	Hyperbola I	0.154654	0.089538		0.25 (15)	2.599 (13)	1.984 (11)	4358 (12)	0.552 (13)	2.939 (10)	2.211 (11)
25	Hyperbola II	0.14431	0.12162	0.02489*	0.247 (12)	2.598 (10)	1.984 (10)	4358 (13)	0.548 (5)	2.953 (15)	2.229 (15)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table E. 2 *Coefficient of equations and the statistical values of Basal Area (*Eucalyptus pellita*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.83659			0.497 (19)	2.888 (19)	2.112 (18)	4550 (19)	0.956 (19)	3.018 (18)	2.272 (18)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.17466	0.7158		0.316 (9)	2.816 (6)	2.001 (5)	4505 (6)	0.757 (10)	2.968 (6)	2.194 (9)
3	Schumacher/ Polymorphic I	3.23032			-0.117 (4)	2.706 (4)	2.041 (14)	4431 (4)	0.098 (2)	2.34 (2)	1.811 (2)
4	**Schumacher/ Polymorphic II**	**3.84725**	**0.58911**		**-0.046 (2)**	**2.512 (1)**	**1.871 (1)**	**4296 (1)**	**0.217 (3)**	**2.36 (3)**	**1.815 (3)**
5	Schumacher /Polymorphic III	3.17573	0.96584		-0.209 (5)	2.695 (3)	2.03 (13)	4425 (3)	0.003 (1)	2.337 (1)	1.806 (1)
6	Schumacher/ Polymorphic IV	1.896161**	-0.011879		-159.447 (23)	2.885 (18)	159.904 (23)	4549 (18)	-134.792 (23)	2.991 (15)	135.362 (23)
7	Johnson–Schumacher	2.62978	0.33033		0.318 (12)	2.818 (9)	2.004 (7)	4506 (8)	0.759 (12)	2.969 (8)	2.194 (6)
8	von Bertalanffy–Richards Anamorphic	0.52316	1.78886		0.313 (7)	2.831 (13)	2.016 (10)	4515 (14)	0.756 (7)	2.981 (14)	2.201 (14)
9	von Bertalanffy – Richards Polymorphic I	32.9326	0.18252		0.062 (3)	2.75 (5)	2.072 (17)	4461 (5)	0.286 (5)	2.584 (5)	2.015 (5)
11	Weibull	0.24	1.465753		0.313 (8)	2.844 (15)	2.027 (12)	4523 (15)	0.757 (9)	2.994 (16)	2.215 (15)
13	Monomolecular	1.15972	0.34732		0.313 (6)	2.818 (9)	2.002 (6)	4507 (9)	0.757 (8)	2.969 (10)	2.194 (8)
14	Richards	1.138	0.2168	2.3373	0.318 (13)	2.817 (8)	1.999 (3)	4507 (10)	0.763 (14)	2.969 (9)	2.2 (12)
15	Gompertz	3.2974	0.77359		0.329 (16)	2.863 (16)	2.057 (15)	4536 (16)	0.774 (16)	3.007 (17)	2.233 (16)
16	Gompertz Polymorphic I	-0.129969	-0.055986		3.255 (22)	5.924 (23)	4.527 (22)	5869 (23)	3.503 (22)	6.173 (22)	4.869 (22)
17	Logistic I	11.26934	1.26058		0.396 (18)	2.957 (20)	2.17 (19)	4595 (20)	0.841 (18)	3.097 (20)	2.337 (19)
18	Logistic II	-1.39861	0.89511		0.337 (17)	2.875 (17)	2.064 (16)	4543 (17)	0.78 (17)	3.025 (19)	2.25 (17)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
19	Logistic III	-2.388202	0.502711		0.581 (20)	3.21 (21)	2.474 (20)	4745 (21)	1.019 (20)	3.348 (21)	2.62 (20)
20	Hossfeld Anamorphic	0.152355	1.509964		2.682 (21)	4.915 (22)	3.467 (21)	5527 (22)	3.113 (21)	6.229 (23)	4.11 (21)
21	Hossfeld Polymorphic	0.03761	-1.55514		-0.011 (1)	2.597 (2)	1.943 (2)	4357 (2)	0.245 (4)	2.507 (4)	1.938 (4)
22	Hossfeld IV	4.8016	1.6692		0.317 (11)	2.826 (12)	2.011 (9)	4511 (12)	0.758 (11)	2.976 (12)	2.197 (10)
23	Levakovic III	8.42432	0.75317		0.319 (14)	2.831 (13)	2.018 (11)	4515 (13)	0.762 (13)	2.979 (13)	2.201 (13)
24	Hyperbola I	0.069473	0.085566		0.323 (15)	2.821 (11)	2.008 (8)	4508 (11)	0.763 (15)	2.97 (11)	2.194 (7)
25	Hyperbola II	0.001039*	0.162149	0.055903*	0.316 (10)	2.816 (6)	1.999 (4)	4506 (7)	0.756 (6)	2.969 (7)	2.199 (11)

*Note: * = some parameters in these equations were insignificant.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table E. 3 *Coefficient of equations and the statistical values of mortality of* Eucalyptus pellita *(using all-possible interval data)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.33838			-19.203 (20)	206.7 (16)	151.4 (14)	12383 (16)	-2.787 (1)	193.2 (19)	148.5 (19)
2	Lundqvist–Korf (Schumacher Anamorphic II)	2.2926*	-0.07783*		-4.334 (7)	201.5 (6)	150.9 (7)	12337 (5)	13.274 (14)	185.6 (5)	142.6 (10)
3	Schumacher/ Polymorphic I	7.03132			-13.482 (17)	248 (20)	194.4 (21)	12717 (20)	18.059 (21)	234.6 (20)	188.1 (21)
4	Schumacher/ Polymorphic II	8.43658	-0.16178		0.463 (1)	199.3 (1)	147.7 (1)	12317 (1)	14.526 (18)	184.4 (1)	141 (2)
5	Schumacher /Polymorphic III	7.004814	0.992925		-16.491 (19)	248 (20)	193.9 (20)	12718 (21)	14.897 (19)	234.7 (21)	187.6 (20)
7	Johnson–Schumacher	-2.0682	2.5937		-4.437 (11)	201.6 (9)	151 (9)	12338 (9)	13.118 (10)	185.7 (8)	142.6 (9)
8	von Bertalanffy–Richards Anamorphic	-0.08494*	-0.17185		-4.202 (5)	201.4 (3)	150.9 (8)	12337 (4)	13.44 (15)	185.6 (4)	142.5 (8)
11	Weibull	0.6369*	-0.2496		-4.385 (9)	201.5 (6)	150.9 (5)	12337 (7)	13.213 (12)	185.7 (7)	142.6 (12)
12	Weibull Polymorphic I	6.75153*	0.02574*	-17.36249*	-0.837 (3)	201.4 (3)	151.4 (15)	12337 (8)	16.462 (20)	185.6 (3)	143.2 (15)
13	Monomolecular	-0.70535	0.46554		-4.52 (12)	201.8 (12)	151.2 (12)	12340 (12)	12.963 (8)	185.8 (11)	142.4 (5)
15	Gompertz	-0.56343	0.38371		-4.416 (10)	201.7 (10)	151.1 (11)	12339 (11)	13.093 (9)	185.8 (10)	142.5 (6)
16	Gompertz Polymorphic I	0.0106773	-0.0011479*		-5.3 (14)	207 (18)	156.4 (19)	12386 (18)	13.829 (17)	189 (16)	142.7 (13)
17	Logistic I	-0.45275	0.30256		-4.328 (6)	201.7 (10)	151.1 (10)	12339 (10)	13.205 (11)	185.7 (9)	142.5 (7)
20	Hossfeld Anamorphic	0.0006893*	-0.1639215*		-13.571 (18)	207 (18)	155.2 (17)	12387 (19)	7.205 (2)	191.1 (17)	147.6 (18)
21	Hossfeld Polymorphic	0.3376*	0.0003026**		-0.633 (2)	199.8 (2)	148.5 (2)	12321 (2)	13.777 (16)	184.7 (2)	141.6 (4)
22	Hossfeld IV	2.0835	-0.2639		-4.365 (8)	201.5 (6)	150.9 (6)	12337 (6)	13.237 (13)	185.6 (6)	142.6 (11)
26	Anamorphic II	-0.10332	-13.86564		-23.135 (21)	206.9 (17)	150.7 (4)	12386 (17)	-9.565 (3)	192.5 (18)	147.1 (17)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
27	Anamorphic III	-0.188657			-4.912 (13)	201.4 (3)	150.7 (3)	12335 (3)	12.608 (6)	185.9 (12)	142.9 (14)
28	Anamorphic IV	-0.0258914			-6.243 (16)	202.1 (13)	151.3 (13)	12341 (13)	11.773 (4)	186.5 (13)	143.6 (16)
29	Polymorphic I	-1575*	0.03795		-3.004 (4)	205.6 (14)	153.5 (16)	12374 (14)	12.951 (7)	187.6 (14)	139.5 (1)
30	Exponential decay anamorphic	0.081265	0		-6.228 (15)	206.5 (15)	155.2 (18)	12381 (15)	12.307 (5)	188.5 (15)	141.2 (3)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table E. 4 *Coefficient of equations and the statistical values of mortality of* Eucalyptus pellita *(using two steps regression: 1 year interval data)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.33347			-38.355 (20)	148.5 (18)	110.7 (18)	4047 (18)	-37.687 (20)	149.9 (18)	113.4 (18)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.3968	-0.4588		-6.582 (6)	136.8 (3)	105.1 (12)	3996 (4)	-0.09 (3)	136.1 (2)	99.5 (11)
3	Schumacher/ Polymorphic I	7.01823			-31.074 (19)	188.9 (20)	150 (20)	4199 (20)	-14.691 (19)	179.4 (20)	139.2 (20)
4	**Schumacher/ Polymorphic II**	**8.34259**	**-0.22143**		**-7.121 (13)**	**135.4 (1)**	**100.6 (1)**	**3990 (1)**	**-5.982 (13)**	**136.2 (10)**	**97.8 (2)**
5	Schumacher /Polymorphic III	7.32559	1.05078		-18.082 (17)	184 (19)	147.5 (19)	4183 (19)	1.6 (9)	172.5 (19)	134.3 (19)
7	Johnson–Schumacher	-8.211**	5.973		-6.676 (10)	136.8 (3)	105 (7)	3997 (7)	-0.179 (6)	136.1 (5)	99.4 (7)
8	von Bertalanffy–Richards Anamorphic	-0.50398	-0.1529		-6.602 (7)	136.8 (3)	105.3 (13)	3996 (3)	-0.119 (4)	136.1 (9)	99.7 (12)
11	Weibull	2.01608	-0.55115		-6.755 (12)	136.8 (3)	105 (8)	3997 (6)	-0.298 (8)	136.1 (7)	99.4 (9)
12	Weibull Polymorphic I	-0.001408*	3.045	-202.6	-0.24 (1)	138.1 (11)	104.8 (4)	4004 (12)	-2.102 (10)	135.1 (1)	93.7 (1)
13	Monomolecular	-1.21621	0.34923		-6.736 (11)	137 (10)	104.9 (5)	3997 (10)	-0.184 (7)	136.1 (8)	99.3 (4)
15	Gompertz	-0.89215	0.23313		-6.627 (8)	136.9 (8)	104.9 (6)	3997 (9)	-0.088 (2)	136.1 (6)	99.4 (6)
16	Gompertz Polymorphic I	0.012199*	-0.004975*		-5.588 (4)	140.9 (16)	109 (16)	4015 (16)	4.036 (11)	139.7 (16)	101.1 (15)
17	Logistic I	-0.66205	0.11782**		-6.556 (5)	136.9 (8)	105 (10)	3997 (8)	-0.028 (1)	136.1 (4)	99.4 (8)
20	Hossfeld Anamorphic	0.0001996	-0.5572		-18.93 (18)	143.8 (17)	110 (17)	4028 (17)	-9.793 (16)	142 (17)	103.6 (17)
21	Hossfeld Polymorphic	0.4383	0.0003138		-8.672 (14)	136.1 (2)	101.5 (2)	3993 (2)	-6.443 (14)	136.7 (11)	98.4 (3)
22	Hossfeld IV	0.3289	-0.7192		-6.648 (9)	136.8 (3)	105.1 (11)	3997 (5)	-0.171 (5)	136.1 (3)	99.4 (10)
27	Anamorphic III	-0.23154			-15.014 (15)	138.5 (12)	104.3 (3)	4003 (11)	-10.847 (17)	138.6 (13)	100.8 (14)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
28	Anamorphic IV	-0.031602			-16.757 (16)	139.4 (13)	105 (9)	4008 (13)	-12.174 (18)	139.3 (15)	101.5 (16)
29	Polymorphic I	-978.72019*	0.07311		-0.27 (2)	139.4 (13)	107.7 (14)	4009 (14)	6.841 (15)	138.2 (12)	99.3 (5)
30	Exponential decay anamorphic	-0.124719			-4.149 (3)	140.2 (15)	108.7 (15)	4011 (15)	5.292 (12)	139.2 (14)	100.8 (13)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table E. 5 *Coefficient of equations and the statistical values of mortality of* Eucalyptus pellita *(using longest interval data)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	-0.3486			2.145 (4)	273.3 (17)	221.2 (18)	2295 (17)	38.379 (18)	264.2 (18)	221.4 (18)
2	Lundqvist–Korf (Schumacher Anamorphic II)	0.2824*	-0.4759*		-3.514 (9)	269.2 (4)	216.3 (13)	2291 (8)	33.13 (8)	258.9 (7)	209.1 (9)
3	Schumacher/ Polymorphic I	7.01547			-3.479 (7)	321.8 (20)	269.1 (20)	2348 (20)	44.119 (20)	303.4 (20)	258.5 (20)
4	Schumacher/ Polymorphic II	9.17574	-0.10011*		3.557 (12)	268.7 (1)	215.5 (4)	2290 (3)	38.2 (17)	259.8 (15)	214 (15)
5	Schumacher /Polymorphic III	6.86778	0.93673		-11.115 (19)	317.1 (19)	262.6 (19)	2344 (19)	41.522 (19)	298.7 (19)	249.2 (19)
7	Johnson–Schumacher	-7.591*	6.884*		-3.522 (10)	269.2 (4)	216.1 (9)	2291 (10)	33.168 (12)	258.8 (4)	209.1 (8)
8	von Bertalanffy–Richards Anamorphic	-0.476*	-0.1173*		-3.459 (6)	269.2 (4)	216.4 (14)	2291 (6)	33.148 (9)	258.9 (8)	209.1 (11)
11	Weibull	2.3456*	-0.4934		-3.529 (11)	269.2 (4)	216.1 (10)	2291 (7)	33.165 (11)	258.8 (5)	209 (7)
12	Weibull Polymorphic I	0.06536*	1.03368*	-79.6224*	-0.447 (2)	269.7 (14)	215.1 (2)	2292 (16)	33.79 (13)	259.4 (9)	209.2 (12)
13	Monomolecular	-0.9089	0.2865*		-3.736 (15)	269.3 (9)	216 (6)	2291 (13)	32.937 (5)	258.8 (1)	208.9 (4)
15	Gompertz	-0.7125	0.204*		-3.657 (14)	269.3 (9)	216.1 (7)	2291 (12)	33.023 (6)	258.8 (2)	209 (5)
16	Gompertz Polymorphic I	0.010324	-0.00476*		-5.779 (16)	270.1 (16)	216.2 (11)	2292 (15)	30.286 (4)	259.8 (12)	208.3 (3)
17	Logistic I	-0.5627	0.122*		-3.596 (13)	269.3 (9)	216.1 (8)	2291 (11)	33.087 (7)	258.8 (3)	209 (6)
20	Hossfeld Anamorphic	0.00008534*	-0.8216*		-14.927 (20)	274.7 (18)	220.9 (17)	2297 (18)	23.947 (1)	262.9 (17)	211.1 (13)
21	Hossfeld Polymorphic	0.2796881*	0.0002178		2.616 (5)	269 (2)	215.9 (5)	2290 (5)	37.889 (16)	259.8 (14)	214.3 (17)
22	Hossfeld IV	0.2387*	-0.6809*		-3.498 (8)	269.2 (4)	216.2 (12)	2291 (9)	33.158 (10)	258.9 (6)	209.1 (10)
27	Anamorphic III	-0.19147			0.181 (1)	269.1 (3)	217.4 (15)	2290 (1)	36.834 (15)	259.8 (13)	213.7 (14)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
28	Anamorphic IV	-0.026332			-1.173 (3)	269.8 (15)	218.2 (16)	2290 (4)	35.945 (14)	260.1 (16)	214.1 (16)
29	Polymorphic I	-845.81579*	0.05331*		-8.956 (17)	269.6 (13)	214 (1)	2291 (14)	25.642 (2)	259.7 (11)	203.1 (1)
30	Exponential decay anamorphic	-0.081265			-10.465 (18)	269.4 (12)	215.3 (3)	2290 (2)	25.971 (3)	259.7 (10)	204.8 (2)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table E. 6 *Coefficient of equations and the statistical values of Dmax (*Eucalyptus pellita*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	1.723			0.352 (19)	1.723 (19)	1.364 (19)	3603 (19)	0.642 (19)	1.818 (18)	1.445 (19)
2	Lundqvist–Korf (Schumacher Anamorphic II)	1.78446	0.42225		0.096 (11)	1.49 (6)	1.101 (6)	3337 (6)	0.364 (12)	1.66 (9)	1.228 (9)
3	Schumacher/ Polymorphic I	3.071849			0.019 (1)	1.435 (5)	1.063 (5)	3268 (5)	0.229 (2)	1.449 (5)	1.095 (5)
4	**Schumacher/ Polymorphic II**	**3.5751**	**0.51098**		**0.03 (4)**	**1.283 (1)**	**0.924 (1)**	**3063 (1)**	**0.256 (4)**	**1.358 (1)**	**1.006 (1)**
5	Schumacher /Polymorphic III	3.03066	0.973287		-0.055 (5)	1.422 (4)	1.049 (4)	3252 (4)	0.155 (1)	1.437 (4)	1.077 (4)
6	Schumacher/ Polymorphic IV	0.959007	0.012393		0.309 (18)	1.719 (18)	1.34 (18)	3599 (18)	0.602 (18)	1.83 (19)	1.443 (18)
7	Johnson–Schumacher	2.37281	0.8527		0.096 (12)	1.493 (9)	1.103 (11)	3341 (9)	0.365 (13)	1.659 (6)	1.228 (7)
8	von Bertalanffy–Richards Anamorphic	0.31979	0.81696		0.093 (7)	1.494 (11)	1.102 (10)	3342 (11)	0.363 (6)	1.661 (12)	1.228 (10)
9	von Bertalanffy – Richards Polymorphic I	23.2281	0.247		0.024 (3)	1.305 (2)	0.925 (2)	3094 (2)	0.241 (3)	1.371 (2)	1.021 (2)
10	von Bertalanffy – Richards Polymorphic II	20.7442	-0.1337		0.021 (2)	1.305 (2)	0.95 (3)	3095 (3)	0.285 (5)	1.389 (3)	1.022 (3)
11	Weibull	0.41095	0.86436		0.094 (9)	1.493 (9)	1.102 (8)	3341 (10)	0.363 (7)	1.661 (11)	1.228 (8)
13	Monomolecular	0.934059	0.375535		0.093 (6)	1.496 (12)	1.105 (13)	3345 (13)	0.363 (8)	1.661 (10)	1.229 (11)
15	Gompertz	1.74854	0.62079		0.099 (14)	1.516 (15)	1.124 (15)	3369 (15)	0.37 (14)	1.675 (15)	1.249 (15)
16	Gompertz Polymorphic I	-0.084769	-0.020849		1.185 (21)	3.323 (21)	2.687 (21)	4809 (21)	1.359 (21)	3.731 (21)	3.023 (21)
17	Logistic I	3.46569	0.87977		0.113 (16)	1.547 (16)	1.158 (16)	3406 (16)	0.386 (16)	1.704 (16)	1.287 (16)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
19	Logistic III	-1.215115	0.396826		0.16 (17)	1.658 (17)	1.278 (17)	3534 (17)	0.436 (17)	1.816 (17)	1.409 (17)
20	Hossfeld Anamorphic	0.09282	0.920097		0.71 (20)	2.354 (20)	1.655 (20)	4177 (20)	1.011 (20)	2.783 (20)	1.962 (20)
21	Hossfeld Polymorphic	1.146468	0.125515		26.137 (22)	516.7 (22)	113.922 (22)	14064 (22)	-123.801 (22)	2582.036 (22)	211.23 (22)
22	Hossfeld IV	2.81775	0.97381		0.095 (10)	1.491 (7)	1.101 (7)	3339 (7)	0.364 (11)	1.66 (8)	1.227 (6)
23	Levakovic III	20.233291	0.354083		0.094 (8)	1.496 (12)	1.105 (12)	3345 (12)	0.363 (10)	1.662 (13)	1.23 (12)
24	Hyperbola I	0.28348	0.098687		0.102 (15)	1.5 (14)	1.11 (14)	3350 (14)	0.37 (15)	1.66 (7)	1.232 (14)
25	Hyperbola II	0.2381	0.24525	0.06812	0.096 (13)	1.491 (7)	1.102 (9)	3341 (8)	0.363 (9)	1.663 (14)	1.231 (13)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Table E. 7 *Coefficient of equations and the statistical values of Dstd (*Eucalyptus pellita*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
1	Schumacher /Anamorphic I	0.8571	0		0.091 (18)	0.571 (19)	0.444 (19)	1577 (19)	0.197 (19)	0.572 (19)	0.443 (19)
2	Lundqvist–Korf (Schumacher Anamorphic II)	1.30697	0.48754		0.062 (15)	0.561 (18)	0.432 (17)	1547 (18)	0.167 (15)	0.557 (17)	0.426 (17)
3	Schumacher/ Polymorphic I	1.21	0		0.002 (3)	0.509 (5)	0.395 (5)	1366 (5)	0.073 (2)	0.523 (5)	0.4 (6)
4	Schumacher/ Polymorphic II	1.45413	0.64225		0.01 (5)	0.499 (3)	0.381 (3)	1332 (3)	0.091 (5)	0.504 (1)	0.388 (3)
5	Schumacher /Polymorphic III	1.16187	0.92034		-0.012 (1)	0.508 (4)	0.393 (4)	1362 (4)	0.059 (1)	0.519 (4)	0.395 (4)
6	Schumacher/ Polymorphic IV	1.32944	-0.31008		0.1 (19)	0.554 (7)	0.433 (18)	1523 (7)	0.196 (18)	0.564 (18)	0.439 (18)
7	Johnson–Schumacher	1.8072	0.7684		0.06 (11)	0.56 (16)	0.431 (15)	1544 (16)	0.165 (10)	0.556 (14)	0.425 (13)
8	von Bertalanffy–Richards Anamorphic	0.39528	0.72626		0.062 (17)	0.559 (13)	0.43 (13)	1541 (14)	0.167 (16)	0.556 (13)	0.425 (14)
9	**von Bertalanffy – Richards Polymorphic I**	**3.74586**	**0.21122**		**0.004 (4)**	**0.493 (1)**	**0.372 (1)**	**1307 (1)**	**0.079 (3)**	**0.505 (2)**	**0.386 (1)**
11	Weibull	0.56696	0.8108		0.062 (16)	0.56 (15)	0.43 (14)	1542 (15)	0.167 (17)	0.556 (15)	0.425 (15)
12	Weibull Polymorphic I	0.76327	-0.68008*		-0.009 (2)	0.545 (6)	0.413 (6)	1494 (6)	0.088 (4)	0.531 (6)	0.399 (5)
13	Monomolecular	0.8665	0.45745		0.062 (13)	0.559 (11)	0.429 (10)	1539 (11)	0.166 (13)	0.555 (11)	0.424 (11)
15	Gompertz	1.4541	0.6598		0.06 (9)	0.557 (10)	0.428 (9)	1534 (10)	0.164 (9)	0.554 (8)	0.423 (8)
16	Gompertz Polymorphic I	-0.13833	-0.03653		0.483 (21)	0.867 (21)	0.69 (21)	2345 (21)	0.577 (21)	0.896 (21)	0.721 (21)
17	Logistic I	2.54648	0.8736		0.059 (8)	0.556 (9)	0.426 (8)	1529 (9)	0.163 (8)	0.553 (7)	0.422 (7)
19	Logistic III	-0.84848	0.40216		0.06 (10)	0.554 (8)	0.424 (7)	1525 (8)	0.165 (11)	0.554 (9)	0.424 (10)
20	Hossfeld Anamorphic	0.32797	1.26269		0.253 (20)	0.684 (20)	0.538 (20)	1909 (20)	0.365 (20)	0.704 (20)	0.554 (20)

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		b	***c***	***d***	**MRES**	**SEE**	**MAE**	**AIC**	**MRES**	**RMSE**	**MAE**
21	Hossfeld Polymorphic	0.270958	-1.11174		0.012 (6)	0.497 (2)	0.379 (2)	1324 (2)	0.096 (6)	0.505 (3)	0.388 (2)
22	Hossfeld IV	1.85381	0.94586		0.062 (14)	0.561 (17)	0.431 (16)	1545 (17)	0.166 (14)	0.557 (16)	0.426 (16)
23	Levakovic III	15.27651	0.30455		0.061 (12)	0.559 (12)	0.43 (11)	1540 (12)	0.166 (12)	0.556 (12)	0.424 (12)
24	Hyperbola I	0.39684	0.12312		0.059 (7)	0.559 (13)	0.43 (12)	1541 (13)	0.163 (7)	0.555 (10)	0.424 (9)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

*Table E. 8 Coefficient of equations and the statistical values of height-dbhob equations (*Eucalyptus pellita*)*

Equation	Model	Coefficients			Statistical values of						
					model fitting				validation		
		a	*b*	*c*	MRES	SEE	MAE	AIC	MRES	RMSE	MAE
4.30	**Naslund**	**0.1252314**	**1.5351449**		**2.32 (9)**	**2.32 (6)**	**1.821 (6)**	**78620 (6)**	**0.026 (9)**	**2.348 (6)**	**1.837 (6)**
4.31	Naslund-Schmidt	1.5722332	0.2016857		0.05 (12)	2.325 (7)	1.825 (7)	78693 (8)	0.05 (12)	2.351 (7)	1.84 (7)
4.32	Naslund 5	1.2788128	0.4695737		0.094 (15)	2.352 (10)	1.854 (10)	79085 (10)	0.097 (15)	2.373 (10)	1.866 (10)
4.33	Curtis	0.941883	-0.117934		-0.037 (11)	2.355 (12)	1.866 (13)	79137 (12)	-0.046 (11)	2.389 (13)	1.884 (13)
4.34	Schumacher	37.27229	10.29748		0.136 (17)	2.401 (15)	1.908 (15)	79803 (15)	0.143 (17)	2.418 (15)	1.917 (15)
4.35	Meyer	-74.34	-0.01552		-0.08 (13)	2.37 (14)	1.896 (14)	79358 (14)	-0.089 (13)	2.405 (14)	1.915 (14)
4.36	Power	0.97443	1.108271		-0.033 (10)	2.354 (11)	1.863 (12)	79116 (11)	-0.041 (10)	2.388 (12)	1.881 (12)
4.37	Michael Menten	0.679997	-2.749852		9.745 (18)	11.77 (18)	10.104 (18)	135096 (18)	9.776 (18)	11.726 (18)	10.049 (18)
4.38	Wykoff	3.744057	12.83014		0.088 (14)	2.355 (12)	1.86 (11)	79139 (13)	0.093 (14)	2.376 (11)	1.872 (11)
4.39	Garcia	37.203	-451.8941		0 (1)	2.586 (17)	2.106 (17)	82388 (17)	0.008 (3)	2.613 (17)	2.121 (17)
4.40	Prodan	-0.013694	1.0243768	-0.7508	-0.111 (16)	2.441 (16)	1.969 (16)	80386 (16)	-0.121 (16)	2.476 (16)	1.988 (16)
4.41	Logistic	22.953695	13.684319	0.2865	-0.009 (3)	2.303 (2)	1.799 (2)	78353 (2)	-0.009 (4)	2.33 (2)	1.816 (2)
4.42	Weibull	23.62	0.009218	1.866	0.025 (8)	2.312 (3)	1.809 (3)	78498 (3)	0.024 (8)	2.34 (3)	1.826 (3)
4.43	Gompertz	27.28595	3.548829	0.15552	0.006 (2)	2.302 (1)	1.797 (1)	78342 (1)	0.005 (1)	2.329 (1)	1.813 (1)
4.44	Sibessen	0.276219	2.351608	0.14948	0.023 (6)	2.325 (7)	1.828 (8)	78691 (7)	0.02 (6)	2.354 (8)	1.845 (8)
4.45	Lundqvist-Korf	290.88061	6.98561	0.34963	0.02 (5)	2.326 (9)	1.83 (9)	78712 (9)	0.017 (5)	2.355 (9)	1.847 (9)
4.46	Ratkowsky	61.968	22.253	4.1276	0.009 (4)	2.314 (4)	1.813 (4)	78530 (4)	0.007 (2)	2.343 (4)	1.83 (4)
4.47	Hossfeld IV	43.15	0.01098	1.588	0.024 (7)	2.315 (5)	1.814 (5)	78548 (5)	0.023 (7)	2.343 (5)	1.831 (5)

*Note: *= insignificant parameter, **= significant at α=0.01. Equations that have insignificant parameter were not chosen.*
Ranking is shown in brackets. (The smallest rank was preferable).

Appendix F Augmented model for all species

Table F. 1 *Standard error for each combination of augmented model by adding elevation, temperature and rainfall for MTH, G and N*

Augmented equation	*Acacia mangium*			*Acacia crassicarpa*			*Eucalyptus pellita*		
	MTH	**G**	**N**	**MTH**	**G**	**N**	**MTH**	**G**	**N**
Original equation	1.315	2.212	112.8	1.363	2.35	134.9	2.013	2.512	135.4
Adding elevation into									
b parameter	1.315*	2.164	**110.2**	1.36#	2.336#	134.6*	2.014*	2.505	135.7*
c parameter	1.315*	**2.14**	-	1.362*	2.33#	134.6*	2.013*	2.504	135.6*
b and c parameter	**1.297**	2.114#	-	1.357	2.33*	134.9*	**1.991**	2.494	135.8*
Adding rainfall into									
b parameter	1.306**	2.201*	111.1	1.363*	**2.242**	134.6*	2.014*	2.498	135.2*
c parameter	1.308#	2.20*	-	1.364*	2.259	133.9*	2.013*	2.502	135.5*
b and c parameter	1.304##	2.195	-	1.361*	2.24*	133.8*	1.999	**2.485**	135.3*
Adding temperature into									
b parameter	1.306#	2.201#	111.3	1.351#	2.333	135.1*	2.012*	2.513*	135.6*
c parameter	1.308#	2.206#	-	1.356#	2.334	135.2*	2.014*	2.514*	135.5*
b and c parameter	1.304**	2.195	-	1.344#	2.33*	**134.3#**	2	2.497	**133.9#**
Adding elevation and rainfall into									
b parameter	1.306*	2.266#	109.4##	1.361*	2.560#	134.4*	2.015*	2.736*	135.3*
c parameter	1.308*	2.258		1.363*	2.564	133.8*	2.014*	2.74*	135.6*
b and c parameter	1.309*	2.249*		1.358*	2.542*	-	1.96*	2.655*	135*
Adding elevation and temperature into									
b parameter	1.306*	2.266	109.5##	1.352*	2.652#	134.9*	2.013*	2.745*	135.9*
c parameter	1.308*	2.258		1.358*	2.614#	134.7*	2.014*	2.741*	135.7*
b and c parameter	1.289*	2.249*		1.344*	2.611*	133.6*	1.974#	-	132.8#
Adding elevation, rainfall and temperature into									
b parameter	1.306*	2.267*	108.2#	1.347*	2.562*	134.4*	2.013*	2.735*	135.1*
c parameter	-	2.259*	-	1.355*	2.561**	-	-	2.741*	-
b and c parameter	-	*	-	1.34*	2.525*	-	1.951*	2.643**	131.9*

*Note : N = stocking before adjustment using probability function. *= insignificant parameters, **=significant at α=0.01, #=insignificant parameters on parameter checking process, ## = significant at α=0.01 on parameter checking process*

Table F. 2 *Standard error for each combination of augmented model by adding elevation, temperature and rainfall for Dmax and Dstd*

Augmented equation	*Acacia mangium*		*Acacia crassicarpa*		*Eucalyptus pellita*	
	Dmax	**Dstd**	**Dmax**	**Dstd**	**Dmax**	**Dstd**
Original equation	2.129	0.4586	2.011	0.5712	1.283	0.4925
Adding elevation into						
b parameter	2.125##	0.4547	2.009**	0.5674#	1.283#	0.4928*
c parameter	2.124#	0.455	2.013*	0.5688#	1.279#	0.4918**
b and c parameter	2.107#	0.4478	1.988##	0.5677*	1.272	0.4822#
Adding rainfall into						
b parameter	2.087#	0.4568**	2.01*	0.5703**	1.297#	0.4927*
c parameter	2.102#	0.457	2.01*	0.5696#	1.293*	0.4926*
b and c parameter	2.068	0.457*	2.011*	0.5698*	1.292*	0.4829#
Adding temperature into						
b parameter	2.087#	0.4568	2.011*	0.5687#	1.281#	0.4899#
c parameter	2.102#	0.457	2.013*	0.5704*	1.278#	0.4892#
b and c parameter	2.07	0.457*	2.001**	-	1.278*	0.4894*
Adding elevation and rainfall into						
b parameter	2.071#	0.4514	2.009*	0.5676*	1.283*	0.493*
c parameter	2.088#	0.4519##	2.012*	0.5684*	1.277**	0.492*
b and c parameter	2.061*	0.4446*	1.988*	0.5653*	1.272*	-
Adding elevation and temperature into						
b parameter	2.071#	0.4514	2.01*	0.5674*	1.276#	0.4891#
c parameter	2.088#	0.4519##	2.014*	0.5692*	1.27#	0.4894*
b and c parameter	2.043#	0.4446*	1.991*	0.568*	1.263*	0.4785*
Adding elevation, rainfall and temperature into						
b parameter	2.071*	0.4516*	2.009*	0.5667*	1.276*	0.4881*
c parameter	-	-	-	-	-	-
b and c parameter	-	-	1.989*	0.5651*	1.264*	-

*Note: *= insignificant parameters, **=significant at α=0.01, #=insignificant parameters on parameter checking process, ## = significant at α=0.01 on parameter checking process*

Table F. 3 *Standard error for each combination of augmented model of Equation 4.43 for all species*

No	Stand Variables					SEE of augmented Gompertz (Equation 4.43)		
	E	**G**	**SI**	**N**	**A**	***Acacia mangium***	***Acacia crassicarpa***	***Eucalyptus pellita***
1	√					1.924	2.463	2.3
2					√	1.253	1.87	1.802
3				√		1.927	2.397	2.284
4		√				1.473	1.968	1.785
5			√			1.889	2.454	2.214
6	√				√	1.252	1.87	1.802
7				√	√	1.25	1.861	1.741**
8		√			√	1.239	1.829	1.687
9		√		√		1.366	1.898	1.783
10			√		√	1.07	1.759	1.544
11	√	√			√	1.239	1.825	1.687
12		√		√	√	1.238	1.828	1.677
13	√			√	√	1.25	1.86	1.741*
14		√	√		√	1.07	1.749	1.542
15	√	√	√		√	1.07	1.745	1.488
16	√	√	√	√	√	1.069*	1.744	1.54

*Note: *= insignificant parameters, **=significant at α=0.01, E = elevation (m), G=basal area (m^2/ha), SI= site index (m), N =stocking (stems/ha), A = age (years).*

Table F. 4 *Coefficients of augmented models*

Variables	Formula		Parameters				
	b	***c***	b_0	b_1	b_2	c_0	c_1
Acacia mangium							
MTH	$b=b_0+b_1*E$	$c=c_0+c_1*E$	43.3479068	-0.0991418		0.088268	0.0013318
G	$b=b_0$	$c=c_0+c_1*E$	3.4677914			0.7156206	0.002135
N	$b=b_0+b_1*E$		-0.1909446	0.0007847			
Dmax	$b=b_0+b_1*R$	$c=c_0+c_1*R$	- 550.2	0.2394		4.415	- 0.001705
Dstd	$b=b_0+b_1*R+b_2*R$	$c=c_0$	-19.137157	0.012758	0.010444	0.13794	
Acacia crassicarpa							
MTH	$b=b_0+b_1*E$	$c=c_0+c_1*E$	30.019985	-0.071118		0.349687	0.00252
G	$b=b_0+b_1*R$	$c=c_0$	4.515	-0.0004855		1.051	
Eucalyptus pellita							
MTH	$b=b_0+b_1*E$	$c=c_0+c_1*E$	21.5169362	0.1292578		0.7038825	-0.0059748
G	$b=b_0+b_1*R$	$c=c_0+c_1*R$	7.499	- 0.001404		- 0.5129	0.00043
Dmax	$b=b_0+b_1*E$	$c=c_0+c_1*E$	17.470623	0.124302		0.474137	-0.004281

Note: R=mean annual rainfall (mm/year), E = elevation (m).